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PRACTICAL GEOMETRY

E. WYNDHAM TARN.

CROSBY LOCKWOOD & CO.

PRACTICAL GEOMETRY

FOR THE

ARCHITECT, ENGINEER, SURVEYOR AND MECHANIC:

Giving Rules for the Delineation and Application of various Geometrical Lines, Figures, and Curves.

BY

E. WYNDHAM TARN, M.A.
ARCHITECT:
AUTHOR OF 'THE SCIENCE OF BUILDING' ETC.

SECOND EDITION,

WITH APPENDICES ON DIAGRAMS OF STRAINS AND ISOMETRICAL PROJECTION.

ILLUSTRATED WITH ONE HUNDRED AND SEVENTY-TWO WOOD ENGRAVINGS.

Capio Lumen

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INTRODUCTION.

THE PRESENT TREATISE is written for the purpose of providing the practical man with some simple rules for the delineation of various geometrical figures and for their application to useful and ornamental purposes. In a work upon 'Practical Geometry' no demonstration is given of the rules laid down, the results only of scientific investigation being explained.

Geometrical figures were undoubtedly made use of for building and other purposes long before any demonstrations or investigations of their properties were attempted; since we find examples in ancient temples of the use of parallels, right-angles, oblongs, squares, circles, conic sections, spirals, and other figures, long before any treatises upon geometry were written to investigate their properties. When, however, logical deduction was brought to bear upon geometry, and it was thereby formed into a 'science,' a large number of valuable properties were discovered that were hitherto unknown, and the modern application of analysis to geometry has greatly extended our knowledge of this science.

In the following pages all abstruse formulæ or complicated methods have been avoided, the rules laid down being rendered as simple as possible so as to enable persons with very moderate knowledge of geometry to work out the various problems and processes with the simplest instruments usually at command.

In the first chapter the various combinations of STRAIGHT LINES are given, the circle being used, however, as a necessary aid to the ready solution of the problems.

The second chapter is devoted entirely to the CIRCLE and the various problems that arise in connection therewith, including its application to architectural purposes as well as to the drawing of polygonal figures.

The curves known as the CONIC SECTIONS are treated in the third chapter; the object being to enable the practical man to apply those curves with the greatest possible facility; and several of their uses in architecture are also suggested. The curve formed by a rope or chain suspended at two points, called the CATENARY, is introduced at the end of this chapter, although in no way connected with the conic sections.

In the fourth chapter some of the curves known to mathematicians as CURVES OF FLEXURE are discussed, and rules given for drawing and applying them to architectural purposes. These curves have the form of the letter S or of the figure 8, and are frequently employed for the contour of mouldings and arches. It is, however, too often the case, that instead of a true mathematical curve being employed, a rough imitation is substituted by drawing two arcs of circles turned in opposite directions and meeting at the line joining their centres. This method is, however, quite contrary to the principle of curves of flexure in which the curvature becomes flatter and flatter as the point of contrary flexure is approached, and merges into a straight line at the actual bending point. Such a curve cannot therefore be imitated by arcs of circles, and it is far better to draw it by hand through 'points' found upon its contour, unless a method of 'continuous motion' can be readily employed.

The fifth chapter is devoted to a large and important class of curves known as SPIRALS. Some few of these can be drawn by 'continuous motion,' but in all cases methods of finding 'points,' and of drawing the curve through those points either by hand or by the compasses are carefully laid down; so that they can be as accurately drawn as is necessary in practice by aid of a drawing board, T square, set squares, and a scale of parts. On account of the frequent use of spirals in decorations, the author has been at considerable pains to make the delineation of them in great variety as easy as possible to those who have but small knowledge of mathematical science.

The sixth chapter explains the methods that may be employed for describing and applying the curves known under the name of CYCLOIDS, some of which may be used in architecture and in the mechanical arts.

There are numerous other curves known to mathematicians which have not been touched upon in the present work, the author having confined himself to those which appeared to him of most practical utility to the class of readers for whom it is intended.

NOTE TO THE SECOND EDITION.

THE present Edition has been enlarged by the addition of Appendices containing a brief notice of the method of applying geometry to the determination of the strains in framework, and of the principles of isometrical projection.

CONTENTS.

CHAPTER I.

THE STRAIGHT LINE.

LINES.

PAGE

1. Definitions of a point, a line, a surface, a plane, a circumference, an angle, a perpendicular 1
2. To divide a right-line of given length into two or more equal parts 2
3. From a given point in a given right-line to erect a perpendicular 3
4. To draw a perpendicular to a given right-line from a given point outside of it. 3
5. From a given point to draw a right-line equal in length to a given line 4
6. From the longer of two unequal right-lines to cut off a part equal to the shorter 5

ANGLES.

7. To construct an angle equal to a given angle 5
8. To divide a given rectilinear angle into two or more equal angles 5
9. To divide a right-angle into three or more equal angles . . 6
10. To divide a right-angle into five or fifteen equal parts . . 7
11. From two given points to draw two right-lines meeting a given right-line in the same point and making equal angles therewith 8

PARALLELS.

12. Definition of parallel lines 8
13. To draw a right-line through a given point and parallel to a given line 9

PROPORTION.

PAGE
14. Definitions of a multiple, equi-multiple, ratio, proportionals, extreme and mean ratio, sum, difference 10
15. To find a third-proportional to two given right-lines . . 11
16. To find a fourth-proportional to three given right-lines . . 11
17. To find a mean-proportional between two given right-lines . 12
18. To divide a right-line of given length into any number of parts . 12
19. To divide a given finite right-line in extreme and mean ratio . 13
20. To produce a given finite right-line so that the whole line thus produced shall be divided in extreme and mean ratio by the end of the given line next the produced part . . . 13
21. Harmonical division of a line 14
22. To divide a given right-line harmonically 14

RECTILINEAR FIGURES.

23. Definitions of figure, perimeter, triangle, quadrilateral, trapezium, parallelogram, rhomboid, rhombus, rectangle, square, regular polygon, similar figures 14
24. To construct an isosceles triangle, of which two unequal sides are given 16
25. To construct an isosceles triangle in which each angle at the base is double that at the vertex 16
26. To construct an equilateral triangle upon a given finite right-line 17
27. To construct a right-angled triangle of which two sides are given 17
28. To construct a triangle whose three sides are of given lengths . 18
29. To construct a triangle of which two sides and the included angle are given 18
30. To construct a triangle of which two angles and a side are given 18
31. To construct a triangle of which two sides and an angle not included are given 19
32. To construct a square upon a given right-line 19
33. In a given triangle to inscribe a square 20
34. To construct a parallelogram of given sides and given angle . 20
35. To construct a square equal in area to the sum of the areas of two given squares 21
36. To construct a square which shall be equal in area to a given parallelogram 21
37. Upon a given right-line to construct a rectangle equal in area to a given rectangle 21
38. To construct a rectangle equal in area to a given irregular rectilinear figure 22

PAGE
39. To construct a rectilinear figure of a given irregular form . 23
40. To divide a given finite right-line into two parts, so that the rectangle under the whole line and one of the parts shall be equal in area to the square upon the other part . . . 23
41. To produce a given right-line so that the rectangle under the whole produced line and the part produced shall be equal in area to the square upon the given line 23
42. To construct a regular pentagon, or figure of five sides, the length of a side being given 24
43. To construct a regular decagon, or figure of ten sides, the length of a side being given 24
44. To construct a regular hexagon, or figure of six sides, the length of a side being given 25
45. To construct a regular octagon, or figure of eight sides, the length of a side being given 26
46. To construct a dodecagon, or figure of twelve sides, the length of a side being given. Also a quindecagon, and polygons of sixteen and of twenty sides 26

CHAPTER II.

THE CIRCLE.

47. Definitions of a curve, circle, tangent, chord, arc, segment, sector, semicircle, quadrant, inscribed circles, concentric circles 29
48. To draw a tangent at a given point of a given circle . . 30
49. To describe a circle of given radius touching a given circle in a given point 30
50. To describe a circle through three given points which are not in the same right-line 31
51. From a given point outside a given circle to draw tangents thereto 31
52. To find any number of points upon an arc of a circle, the middle point and two extremities of the arc being given . . . 33
53. To draw an arc of a circle by continuous motion, when the centre is not available 34
54. On a given right-line to describe a segment of a circle that shall contain a given angle 35
55. From the extremities of a given right-line to draw two other lines in any directions meeting at right-angles to each other . 35

PAGE

56. To lay out an arc of a circle as a railway curve of given radius, by finding points on the circumference 36
57. To find arithmetically the radius of an arc of a circle of which the chord and height are given 36
58. To describe a circle touching a given right-line and passing through two given points 37
59. To describe a circle touching a given circle, and also a given right-line outside of it in a given point 38
60. To describe a circle of given radius which shall touch two given circles 39
61. To describe a circle through two given points and touching a given circle 41
62. To draw geometrical tracery in a Gothic arch 43
63. To describe a circle which shall touch a given circle and a given right line, and shall also have its centre on another given line 45
64. To describe a circle touching a given circle, and also two given right-lines which are not parallel 46

INSCRIBED AND CIRCUMSCRIBED CIRCLES.

65. To inscribe three circles in an equilateral Gothic arch . . 46
66. To inscribe three circles in an equilateral triangle . . . 48
67. To inscribe three circles in a given isosceles triangle . . 48
68. In a given triangle to inscribe a circle 49
69. In an equilateral triangle to inscribe a circle 49
70. In a given square to inscribe a circle 50
71. In any regular polygon to inscribe a circle 51
72. In a given square to inscribe four equal circles . . . 51
73. In any given regular polygon to inscribe as many equal circles as the figure has sides 51
74. In a given circle to inscribe three equal circles 52
75. To inscribe six equal circles in a given circle 53
76. To inscribe twelve equal circles in a given circle 53
77. To inscribe four equal circles in a given circle 54
78. To inscribe eight equal circles in a given circle 55
79. In a given circle to inscribe five equal circles 56
80. In a given circle to inscribe ten equal circles 56
81. In a given circle to inscribe (approximately) seven equal circles 57
82. To circumscribe a circle about a given triangle 58
83. To circumscribe a circle about any given regular polygon . 58

INSCRIBED AND CIRCUMSCRIBED POLYGONS.

PAGE
84. In a given circle to inscribe a triangle similar to a given triangle 58
85. About a given circle to circumscribe a triangle similar to a given triangle 59
86. In a given circle to inscribe an equilateral triangle . . . 60
87. About a given circle to circumscribe an equilateral triangle . 60
88. In a given circle to inscribe a square 61
89. About a given circle to circumscribe a square . . . 61
90. In a given circle to inscribe a regular octagon or figure of eight equal sides 61
91. About a given circle to circumscribe a regular octagon . . 61
92. In a given circle to inscribe a regular pentagon or decagon . 61
93. About a given circle to circumscribe a regular pentagon or decagon 62
94. In a given circle to inscribe a regular hexagon, or figure of six equal sides 63
95. About a given circle to circumscribe a regular hexagon . . 63
96. To inscribe in a given circle a regular dodecagon, or figure of twelve equal sides 63
97. About a given circle to circumscribe a regular dodecagon . 64
98. In a given circle to inscribe a quindecagon, or figure of fifteen equal sides 64
99. In a given circle to inscribe a polygon of any number of equal sides 65
100. About a given circle to circumscribe a polygon of any number of equal sides 66
101. To measure the curvature of curved lines 66

CHAPTER III.

CONIC SECTIONS.

102. Definitions.—Ellipse, parabola, hyperbola 68

THE ELLIPSE.

103. Definitions.—Major and minor axes, centre, focus, radius-vector, eccentricity, tangent, normal 69
104. To describe an ellipse of given length of axes by continuous motion 70

PAGE
105. To find any number of points on the circumference of an ellipse of given axes 71
106. To draw an ellipse by help of the circumscribed circle, the length of the axes being given 71
107. To draw a tangent at any given point on an ellipse, by help of the foci 72
108. To draw a tangent at any point of an ellipse, by help of the circumscribed circle 72
109. To draw a tangent to an ellipse from a given point on the major-axis produced 72
110. To draw the normal, or perpendicular to the tangent at any given point on an ellipse 73
111. To draw tangents to an ellipse from any given point outside of it 73
112. About a given ellipse to circumscribe a trapezium of which two opposite vertices are given 74
113. To approximate to the outline of an ellipse by means of arcs of circles 74
114. Definitions.—Diameter, conjugate-diameter, ordinate, supplemental chords 75
115. A pair of conjugate-axes being given, to find points on the ellipse by means of the supplemental chords . . . 76
116. In a given parallelogram to inscribe an ellipse . . . 77
117. Any pair of conjugate-diameters of an ellipse being given, to find the axes 77
118. To apply the ellipse to form the contour of architectural mouldings 78
119. To apply the ellipse to form the contour of leaf ornaments . 81
120. To apply the ellipse to form the entasis of a column . . 81
121. To apply the ellipse to form the outline of a Tudor arch . 82
122. A diameter of an ellipse and an ordinate being given, to find the conjugate-diameter 83
123. To inscribe an ellipse in a given trapezium 84

THE HYPERBOLA.

124. Definitions.—Hyperbola, vertex, axis, foci, centre, eccentricity, radius-vector, asymptotes, rectangular-hyperbola, latus-rectum, abscissæ 85
125. To draw a hyperbola by continuous motion 86
126. To find any number of points upon the contour of a hyperbola, whose vertex and foci are given 87
127. To draw the asymptotes of a hyperbola, whose vertex and foci are given 87

PAGE
128. To find points on a hyperbola by means of the asymptotes . 88
129. To draw any hyperbola by means of the rectangular hyperbola 89
130. To draw any hyperbola by means of a scale of parts . . 90
131. To find points on the contour of a rectangular hyperbola . 92
132. To find points on the contour of any hyperbola by the ruler only 92
133. To draw the tangent at any given point of a hyperbola . . 93
134. To draw a tangent to a hyperbola from any point upon the axis outside the curve 93
135. To draw a normal at any given point on a hyperbola . . 94
136. To find the foci of a given hyperbola 94
137. To approximate to the contour of a hyperbola by means of arcs of circles 94
138. To apply the hyperbola to form the entasis of a column . 95
139. Application of the hyperbola to form the outline of mouldings and other architectural forms 95

THE PARABOLA.

140. Definitions.—Parabola, axis, vertex, latus-rectum, directrix, radius-vector, abscissa, ordinate 97
141. To draw the parabola by continuous motion 98
142. To find points on the contour of the parabola 99
143. To find the lengths of ordinates to a parabola by means of a scale of parts 99
144. To draw a tangent at any point of the parabola . . . 100
145. From a given point on the axis outside the parabola to draw a tangent 101
146. To draw a normal at any given point on the parabola . . 101
147. To apply the parabola to form the entasis of a column . . 101
148. To find the focus of a given parabola 102
149. To apply the parabola to form the outline of a Tudor arch . 102
150. To approximate to the contour of the parabola by arcs of circles 103
151. Application of the parabola to mouldings, architectural ornaments, and other purposes 104

THE CATENARY.

152. Definition of a catenary 106
153. To apply the catenary to form the entasis of a column . . 106
154. To apply the catenary to form the entasis of a spire . . 106

CHAPTER IV.

CURVES OF FLEXURE.

PAGE
155. Definitions.—Curve of flexure, point of contrary-flexure, harmonic-curve or curve of sines, lemniscate . . . 108

THE HARMONIC-CURVE.

156. To find points on the contour of the harmonic-curve . . 108
157. To find points on the harmonic-curve by a scale of parts . 110
158. To draw the tangent at any point of the harmonic-curve . 111
159. To draw the normal at any given point of the harmonic-curve 112
160. To set out the harmonic-curve as a railway curve . . 112
161. To apply the harmonic-curve to form the entasis of a column 113
162. To apply the harmonic-curve to form the contour of an ogival arch or pinnacle 114
163. To apply the harmonic-curve to mouldings and other architectural ornaments 115

THE LEMNISCATE.

164. To draw a lemniscate by continuous motion. 116
165. To find points on the contour of a lemniscate . . . 117
166. To find the algebraical equation to the lemniscate . . 118
167. To draw a lemniscate of given dimensions 118
168. To draw a tangent to the lemniscate 119
169. To apply the lemniscate to form an ogival arch . . . 120
170. To apply the lemniscate to mouldings and other architectural ornament 121

CHAPTER V.

SPIRALS.

171. Definitions.—Pole, spiral, radius-vector, axis, involute of circle, spiral of Archimedes, reciprocal spiral, equiangular or logarithmic spiral 124

THE INVOLUTE OF THE CIRCLE.

172. To draw the involute by continuous motion. 127
173. To find points upon the involute 128

THE SPIRAL OF ARCHIMEDES.

PAGE
174. To draw a spiral whose distance from the pole is directly proportional to the angle made by the radius-vector with the fixed axis 129
175. To draw the tangent at various points of the foregoing spiral . 131
176. To find points on the spiral whose distance from the pole is proportional to the square of the angle made by the radius-vector with the fixed axis 131
177. To draw the tangents at various points of the foregoing spiral 133
178. To find points on the spiral whose distance from the pole is proportional to the cube of the angle made by the radius-vector with the fixed axis 134
179. To draw the tangents at various points of the foregoing spiral 137
180. To apply the above spiral to form a volute of given dimensions 138
181. Table of the squares and cubes of integers 140

THE RECIPROCAL SPIRAL.

182. To find points on the spiral in which the length of the radius-vector varies inversely as the angle of revolution from a fixed axis 141
183. To draw tangents at various points on the reciprocal spiral . 144

THE LITUUS.

184. To find points on the contour of the spiral in which the radius-vector varies inversely as the square root of the angle of revolution from a fixed axis 146
185. To draw tangents at various points of the foregoing spiral . 148
186. To apply the lituus to form the outline of a console or double volute 149
187. To find points upon a spiral in which the radius-vector varies inversely as the cube-root of the angle of revolution from a fixed axis 151
188. To draw tangents at various points of the foregoing spiral . 153

THE EQUIANGULAR OR LOGARITHMIC SPIRAL.

189. To find points on the equiangular spiral, the depth and axis of which are given 154
190. To draw tangents and normals at various points on the equiangular spiral. 156

PAGE
191. To draw the equiangular spiral when the depth and the angle between the tangent and radius are given . . . 157
192. To apply the equiangular spiral to the formation of volutes and other architectural ornaments 162

CHAPTER VI.

CYCLOIDS.

193. To describe a cycloid by continuous motion 165
194. To find points upon the perimeter of the cycloid . . . 165
195. To draw the tangent and normal at any given point on the cycloid 166
196. Properties and applications of the cycloid 167
197. To describe the curve called the companion to the cycloid . 167
198. Definition of a trochoid 168
199. To find points upon a trochoid 168
200. To describe an epicycloid and hypocycloid 169

APPENDIX A.

DIAGRAM OF STRAINS.

201. To determine geometrically the relative strains in the parts of a frame 171

APPENDIX B.

ISOMETRICAL PROJECTION.

202. To draw a view of a group of buildings 176

PRACTICAL GEOMETRY.

CHAPTER I.

THE STRAIGHT LINE.

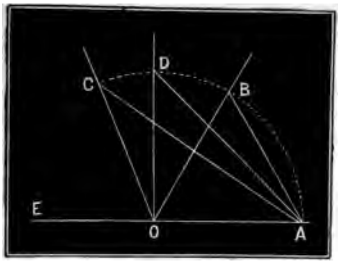
Lines.

1. Definitions.—A *mathematical point* is a position in space of which no part can be taken; a *mathematical line* is formed by the motion of a point in space; when the motion is always in the *same* direction, a *right* or *straight line* is produced. A *physical line* is drawn by the motion of the point of a pen or pencil over any surface. A *mathematical surface* is produced by the motion of a line in any direction; when the moving line is straight and the motion all in one direction, the surface is called a *plane*. When a portion of a surface is entirely enclosed by a line or lines, the boundary is called the *perimeter* or *circumference*. When two right lines meet together at a point they are said to form an *angle* with each other, which is greater or less according to the amount of turning or inclination which one line makes with the other; for example, the angle A O C (fig. 1) which O C makes with O A is greater than the angle A O B which B O makes with O A. Let a circle be drawn from O as a centre and with any radius, then the angle at O is measured by the arc which it subtends, the arc and the angle increasing and decreasing together; when

4.

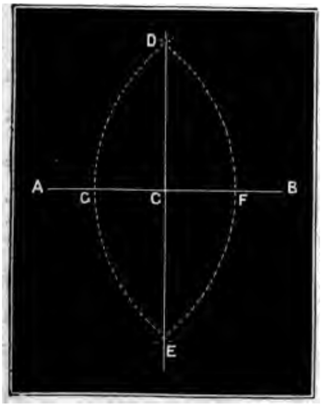
the angle C O A is double the angle B O A, the arc A C is twice as long as the arc A B. If the line A O is produced to any point E, and the *perpendicular* O D is set up so as to make the angle A O D equal to the angle E O D, then each of these angles is called a *right angle.* The angle A O B, which is less than a right angle is called an *acute angle*, and the angle A O C, which is greater than a right angle, is called an *obtuse angle.* An angle is said to be *included* by the two lines forming it. The point O is called the *vertex* or *angular point* of the angles A O C, A O D, A O B.

Fig. 1.



2. *To divide a right line of given length into two or more equal parts.*—Let A B (fig. 2) be the given line. From the points A and B as centres, and with any radius, B G or A F, greater than half the length of A B, describe circles cutting each other in D and E; then the straight line D C E will bisect A B in C, making C A equal to C B; also D C will be perpendicular to A B, the angles A C D and B C D being equal.

Fig. 2.



If we bisect each of the equal parts A C and B C in the manner above described, we shall have the line A B divided into four equal parts; and if each of these is again bisected the line will be divided into eight equal parts. In this manner we can divide a line into any number of equal parts, that number being a power of two, as 4, 8, 16, 32, and so on.

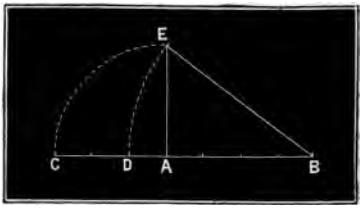
When the given line is too long to allow of the arcs being struck from the two ends, its middle point can be found by measuring equal distances A G and B F from each end, and then bisecting, by the foregoing method, the line between the points F, G in the point C which will also be the middle point of the given line A B.

3. *From a given point in a given right line to erect a perpendicular.*—Let C (fig. 2) be the given point in the given line A B. From C as a centre and with any radius C A describe a circle cutting the line in A and B. From A and B as centres, and with any radius A F greater than A C, describe circles cutting each other in the points D and E; then the right line D C E will be perpendicular or at right angles to A B.

Another method.

Let C A B (fig. 3) be the given line, A the given point. Take any length A D and measure A C equal to three times A D, and A B equal to four times A D. From A as a centre, and with A C for radius, describe a circle; from B as a centre and with B D as a radius, draw an arc cutting the circle in the point E. Then the straight line E A is perpendicular to A B.

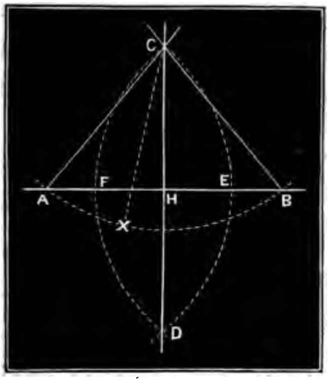
Fig. 3.



This method is used by surveyors in setting out a right angle; let a cord or measuring tape be held tight at three points so as to form a triangle A B E, having the lengths of its sides in the proportion of 3, 4, and 5; then the angle B A E, which is opposite the longest side or *hypotenuse* B E, will be a right angle.

4. *To draw a perpendicular to a given right-line from a* *given point outside of it.*—Let A B (fig. 4) be the given line, C the given point. Take any point X on the opposite side of the line to that on which C lies, and join C X. From C as a centre, and with C X for radius, describe a circle cutting the given line in the points A and B; join A C, B C. From A and B as centres, with A C as radius, describe circles intersecting in C and D; draw C D cutting A B in the point H, then the line C H D is perpendicular to A B.

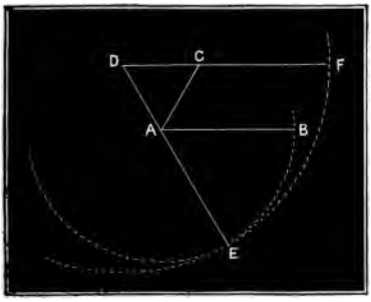
Fig. 4.



In applying this process to surveying, it is only necessary to measure two equal lengths from C to A B, namely, C A and C B, and to bisect the line A B in H; then the line H C will be perpendicular to A B.

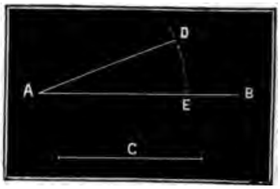
5. *From a given point to draw a right line equal in length to a given line.*—Let A B (fig. 5) be the given line, C the given point. Join A C. From A and C as centres, and with A C for radius, describe circles cutting each other in the point D; join D C and D A. From A as a centre, and with A B as a radius, describe a circle and produce D A to meet it in E. From D as a centre, and with D E for radius, describe a circle, and produce D C to meet it in F; then C F is equal to A B, and is drawn from the given point C.

Fig. 5.



6. *From the longer of two unequal right lines to cut off a part equal to the shorter.*—Let A B (fig. 6) be the longer of the two lines, from which it is required to cut off a part equal to C the shorter. From the point A draw in any direction the line A D equal to C (**5**). From A as a centre, and with radius A D, describe a circle cutting A B in E; then A E is equal in length to C.

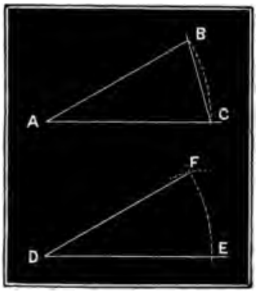
Fig. 6.



ANGLES.

7. *To construct an angle equal to a given angle.*—Let B A C (fig. 7) be the given angle made by the two lines A B and A C meeting at A. From A as a centre and with any radius A B, describe a circle cutting A B and A C in B and C; join B C. Draw any right line D E, and from D as a centre, with a radius equal to A B, describe a circle cutting D E in the point E. From E as a centre, with a radius equal to B C, describe an arc cutting the circle in the point F. Join F D; then the angle F D E is equal to the angle B A C. If D E is parallel (**12**) to A C, then D F will be also parallel to A B.

Fig. 7.

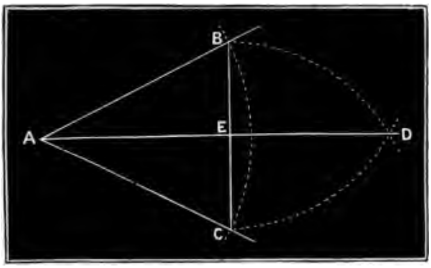


8. *To divide a given rectilinear angle into two or more equal angles.*—Let B A C (fig. 8) be the given angle made by the lines A B and A C. From A as a centre, with any radius A B, describe a circle cutting the lines in B and C; join B and C. From B and C as centres, and with B C as a radius, describe circles intersecting at the point D.

Join A D; then the angle B A C is bisected or divided into two equal angles by the line A D.

In practice it is sufficient to bisect the line B C (**2**) in the point E, and the line E A bisects the given angle.

Fig. 8.

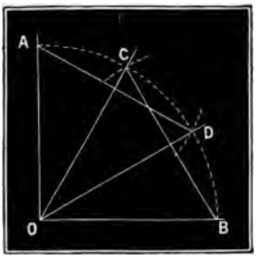


If we bisect in the same manner each of the angles B A E, C A E, then the given angle B A C will be divided into four equal angles, and as each of these again can be bisected, the angle will be divided into eight equal angles; and in this way we can subdivide an angle into any number of equal angles, that number being a *power* of two, as 4, 8, 16, 32, and so forth.

We cannot divide an angle *mathematically* into any odd number of equal parts, but *practically* this can be done as nearly as we please by dividing the arc B C by means of dividers, or by the use of an instrument called a protractor.

9. *To divide a right angle into three or more equal angles.*—Let A O B (fig. 9) be the given right angle. From O as a centre, and with any radius O A, describe a circle; and from A and B as centres, with the same radius, draw arcs, cutting the circle in the points C and D. Join C O and D O; then the angles A O C, C O D, D O B are all equal.

Fig. 9.

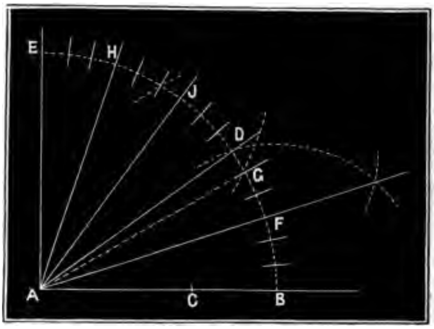


Also, since we can bisect each of the angles A O C, C O D, D O B (**8**), and divide them into any number of equal parts being a power of 2; we are enabled to divide a right angle into 3, 6, 12, 24, 48 equal parts, or any number being a product of 3 and a power of 2.

The half, quarter, eighth, and so on, of a right angle can be subdivided in the same way, for one-sixth of a right angle is one-third of half a right angle; and one-twelfth of a right angle is one-third of a quarter of a right angle; and so forth.

10. *To divide a right angle into five or fifteen equal parts.*—Let the line A E (fig. 10) be at right angles to A B. Take any length A B, and divide it at C in *extreme and mean ratio* (**19**). From A as a centre, with radius A B, describe a circle, and from B as a centre, with A C for radius, draw an arc cutting the circle in D. Join D A, then the angle D A B is one-fifth of two right angles, or two-fifths of one right angle; bisect the angle D A B (**8**) by the line A F, and the angle B A F is one-fifth of the right angle E A B. Divide the angle F A E into four equal parts by the lines D A, J A, H A, and the right angle will be then divided into five parts, each equal to F A B.

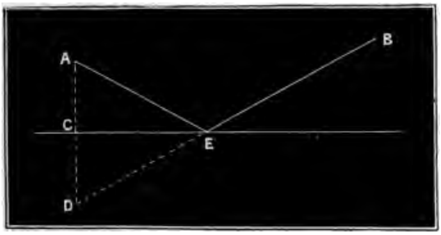
Fig. 10.



To divide the angle B A E into fifteen equal parts: draw the circle B D E from A as a centre; then from E as a centre, with E A for radius, draw an arc cutting the circle at G; join G A. Then G A B is one-third or five-fifteenths of a right angle, and D A B being two-fifths or six-fifteenths of a right angle, consequently the angle G A D is one-fifteenth of the right angle. By measuring D G along the quadrant B E, we can divide it into fifteen equal parts, and hence the right angle can also be divided into fifteen equal parts. Since each of the five or fifteen parts can be divided by two, four, eight, &c., it is possible to divide the right angle into five, ten, fifteen, twenty, thirty, and any other number of equal parts, being a product of five or fifteen into a power of two.

11. *From two given points to draw two right lines meeting a given right line in the same point and making equal angles therewith.*—Let A and B (fig. 11) be the given points, C E the given line. If A and B are on the same side of the given line, then draw from one of them, as A, the line A C at right angles to C E (**4**); produce A C to D, making C D equal to C A. Draw D B, cutting the given line at E, and join A E; then the lines A E and B E make equal angles at E, with the line C E.

Fig. 11.



If the given points, as B and D, are on opposite sides of the given line, draw D B cutting the line at E, then D E and B E make equal angles at E with the given line.

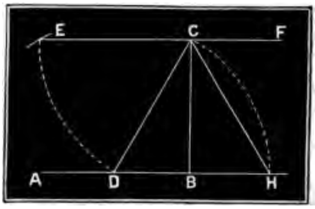
PARALLELS.

12. DEFINITIONS.—Two right lines which lie in the same plane are said to be parallel to one another when their distance apart is everywhere the same, so that however far they may be produced in either direction they will never meet.

If two lines are each parallel to a third line, they are parallel to one another. If two lines which are parallel are also of equal length, the right lines which join the adjacent extremities of those lines will be themselves equal and parallel. If the lines A B and E F (fig. 12) are parallel and any line C D is drawn across them, then the angles E C D, C D B are equal, and the angles A D C, D C F are also equal; these angles are called *alternate* angles. The sum of the two angles F C D, C D B is equal to two right angles, and the sum of the angles E C D, C D A is also equal to two right angles.

13. *To draw a right line through a given point and parallel to a given line.*—Let A B (fig. 12) be the given line, C the given point. Draw any right line C D, cutting A B in D; and from D and C as centres, with D C as radius, describe circles C H and D E. Let the arc C H cut A B in H, and join C H. Then from D as a centre, and with H C as radius, describe an arc cutting the circle D E in E. Draw the right line C E, which will be parallel to the given line A B.

Fig. 12.



Another method is to draw C B perpendicular to A B (4), and then to draw E C perpendicular to C B; E C will be parallel to A B. This is a rule that is readily available for setting out parallel lines upon the ground.

When an instrument for setting out angles is used, we have only to make the angle E C D equal to the angle C D H, and the lines E C and A B will be parallel.

The line C B perpendicular to both the parallels represents the shortest distance between them, which is always of the same length for the same pair of parallels.

PROPORTION.

14. DEFINITIONS.—A greater magnitude is said to be a *multiple* of a lesser one of the same kind, when the greater contains the less a certain number of times exactly. Thus the numbers 4, 6, 8, &c., are multiples of 2. *Equi-multiples* of two different quantities are when those quantities are taken the same number of times; thus, the numbers 4 and 6 are equi-multiples of 2 and 3, 4 being twice 2, and 6 twice 3.

Ratio, *proportion*, or *relative magnitude*, is a mutual relation of two magnitudes of the same kind to one another, with respect to the number of times that one is contained in the other. If there be four magnitudes, and any equi-multiples whatsoever of the first and third be taken, and also any equi-multiples whatsoever of the second and fourth be assumed; if, according as the multiple of the first is greater, equal to, or less than the multiple of the second, the multiple of the third is also greater, equal to, or less than the multiple of the fourth; then the first is said to have to the second the same ratio or proportion as the third has to the fourth.

Magnitudes which have the same ratio to one another are called *proportionals*. If there are three quantities, and the first has to the second the same ratio as the second has to the third, then the last is called a *third-proportional* to the first and second, and the second is called a *mean-proportional* between the first and third. If there are four quantities, and the first has to the second the same ratio as the third has to the fourth, then the last is called a *fourth-proportional* to the rest, and the second and third are two *mean-proportionals* between the first and fourth.

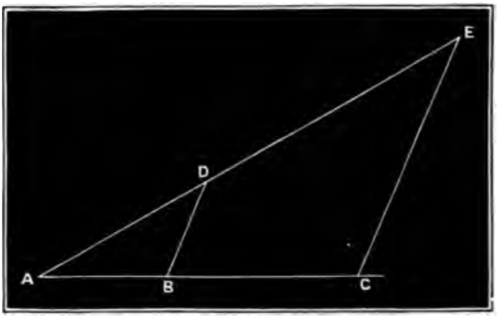
When four magnitudes are proportional, it is usual to say that the first is to the second as the third to the fourth; or, the first contains, or is contained in, the second the same number of times, or parts of a time, as the third contains, or is contained in, the fourth.

When a right line A B (fig. 10) is divided into two unequal parts at C, so that the ratio of the whole line A B to the larger part A C is the same as that of the larger part A C to the lesser part C B, the line is said to be cut in *extreme and mean ratio* by the dividing point C.

The *sum* of two or more quantities is the quantity obtained by adding them together. The *difference* of two quantities is the amount by which one exceeds the other.

15. *To find a third-proportional to two given right lines.*—From any point A (fig. 13) draw two lines A C, A E, making an acute angle with each other. On A C take A B equal to the first and B C to the second of the given lines. On A E take A D equal to B C. Join B D, and draw C E parallel (**13**) to B D; then D E is the *third-proportional* to A B and A D, or, A B is to A D (or B C) as A D is to D E.

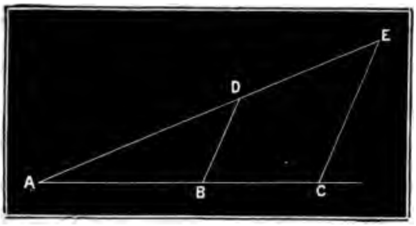
Fig. 13.



16. *To find a fourth-proportional to three given right lines.*—From any point A (fig. 14) draw two right lines A C and A E, making an acute angle at A. Take A B equal to the first of the given lines, B C to the second, and

A D to the third. Join B D, and draw C E parallel (**13**) to B D. Then D E is the *fourth-proportional* to the lines A B, B C, A D; or, A B is to B C as A D to D E, which is generally written thus:

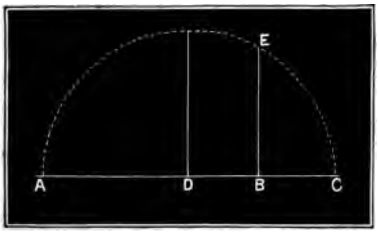
Fig. 14.



A B : B C :: A D : D E.

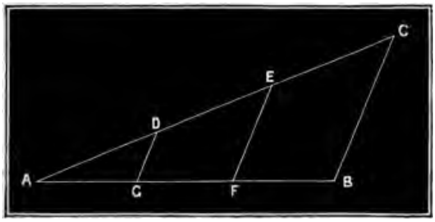
17. *To find a mean-proportional between two given right lines.*—Draw any straight line A C (fig. 15), and on it take A B and B C equal to the two given lines. Bisect A C in the point D, and from D as a centre, with D A as radius, describe a circle. At B erect the line B E perpendicular to A C (**3**), meeting the circle in the point E. Then B E is a *mean-proportional* between A B and B C; or, A B : B E :: B E : B C.

Fig. 15.



18. *To divide a right-line of given length into any number of parts.*—Let A B (fig. 16) be the given line. From A draw a line A C making any angle with A B. Measure by any scale the lengths A D, D E, E C, in the proportion of the required divisions. Join C B, and draw E F, D G, parallel to C B; then the line A B is divided at G and F similarly to the line A C, and the divisions A G, G F, F B have the same ratio to one another as A D, D E, and E C have to each other.

Fig. 16.



In this way the line A B can be divided into any number of equal or unequal parts, by setting off on the line A C the same number of parts in the proportions required. If the divisions on A C are all equal, then those on A B will also be equal to one another; if the divisions on A C are in any such proportion as the numbers 2, 3, 4, then those on A B will bear the same proportion to each other.

19. *To divide a given finite right-line in extreme and mean ratio.*—Let A B (fig. 17) be the given line; bisect it in C. Draw A E perpendicular to A B (**3**), and make A E equal to A C. Join E B, and produce E A to D, making E D equal to E B. On A B cut off A F equal to A D, and the point F will divide the line A B as required; or, we have:

A B : A F : : A F : B F.

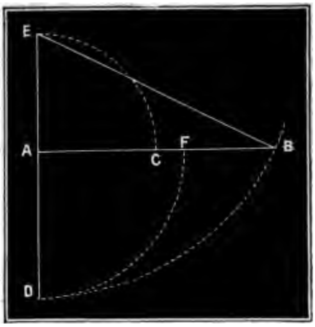


Fig. 17.

20. *To produce a given finite right-line so that the whole line thus produced shall be divided in extreme and mean ratio by the end of the given line next the produced part.*—Let A B (fig. 18) be the given line, bisected at C. Draw C D at right angles to A B, and make C D equal to A B. Join A D, and produce A B to E, making C E equal to A D. Then the line A E is divided in *extreme and mean ratio* by the end B of the given line A B; or, we have:

A E : A B : : A B : B E.

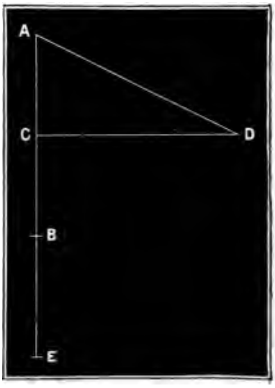
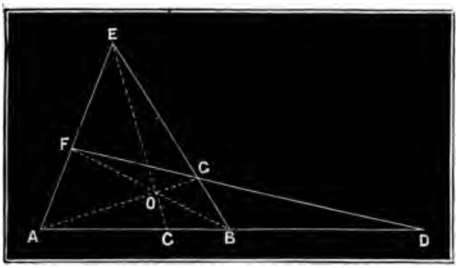


Fig. 18.

21. Definitions.—A given right line, as A B (fig. 19) is said to be *harmonically* divided, when two points C and D, one within the line, and the other in the line produced, are so placed that A C : C B : : A D : DB. C D is called an *harmonic mean* between A D and B D.

22. *To divide a given right-line harmonically.*—Let A B (fig. 19) be the given line divided at the point C; it is required to find the point D, in A B produced, so placed that A D : D B : : A C : C B.

Fig. 19.



On A B construct any triangle A E B, and join E C. Take any point O upon E C, and draw B O F, cutting A E in F. Draw A O G cutting B E in G. Through F and G draw a line cutting A B produced at the point D; then A D : D B : : A C : C B; or A B is *harmonically* divided by C and D.

Rectilinear Figures.

23. Definitions.—A *figure* is a portion of surface enclosed on all sides by a line or lines, which is termed the *boundary* or *perimeter.* When the boundary consists of straight lines it is a *rectilinear figure*, which cannot be formed by less than three straight lines. A plane surface enclosed by three right lines is called a *triangle* or *trilateral figure*: a triangle which has two equal sides is termed *isosceles*; and if all three sides are equal it is an *equilateral triangle*; when one of the angles is a right angle it is called a *right-angled* triangle, as A O D (fig. 1); if one angle is greater than a right angle, it is an *obtuse-angled* triangle, as A O C (fig. 1); and if each of the angles is less than a right angle, it is an *acute-angled* triangle, as A O B (fig. 1). No triangle can have more than one right angle, and the sum of the three angles of every triangle is always equal to two right angles. In every triangle the sum of the lengths of any two sides is greater than the length of the third side.

A figure formed by four right lines is termed a *quadrilateral figure*, and the straight lines which join the opposite corners are called *diagonals*, as A D and B C (fig. 29), which are the diagonals of the figure A B D C. An irregular quadrilateral figure, having all the sides of different lengths, is called a *trapezium.* A *parallelogram* or *rhomboid* is a quadrilateral figure, having the opposite sides equal and parallel, as A B D C (fig. 29), and if all the sides are equal it is called a *lozenge* or *rhombus.* A quadrilateral figure in which all the angles are right angles is termed a *rectangle* or *oblong*, as A B F E (fig. 29), and when all the sides are also equal it is a *square* (fig. 27). A rectangle whose adjacent sides are equal to two given lines is called the rectangle *under* those lines, thus A B F E (fig. 29) is the rectangle under A B and A E. A square having its side equal to a given line A B (fig. 27), is called the square *upon* A B.

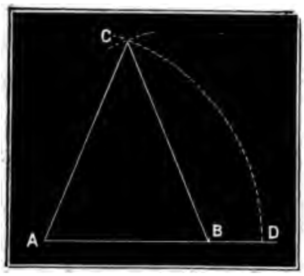
A *regular polygon* is a figure having all its sides equal, and all the angles made by those sides with each other also equal, so that a circle can be drawn through all its angular points.

The area or amount of surface contained in any figure is measured by the number of square units, or squares of a given size, which it covers. The finding of areas of various figures belongs properly to the science of 'Mensuration.'

Two rectilinear figures are said to be *similar* when the angles in one are severally equal to the angles in the other and the sides about the equal angles are proportional; thus the triangles B A D and C A E (figs. 13 and 14) are similar.

24. *To construct an isosceles triangle, of which two unequal sides are given.*—Draw a straight line, A B D (fig. 20), and take A B equal to the given base of the triangle, A. D equal to one of the given equal sides. From A and B as centres, and with A D as radius, describe circles cutting each other in the point C; join A C, B C. Then A C B is the isosceles triangle required, having its two legs A C and B C equal to one another.

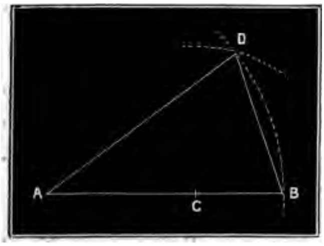
Fig. 20.



25. *To construct an isosceles triangle in which each angle at the base is double that at the vertex.*—First, when the length of the equal sides is given.

Let A B (fig. 21) be the given length of one of the sides. Divide A B in *extreme and mean ratio* (**19**) at the point C. From A as a centre, and with A B for radius, describe a circle; and from B as a centre, with A C for radius, draw an arc cutting the circle in the point D. Join A D, B D; then B A D is the triangle required, each angle at B and D being double the angle at A.

Fig. 21.

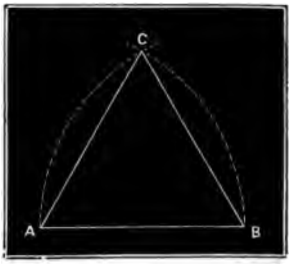


Secondly, when the length of the base only is given.

Let A C be the given length of the base. Produce A C to B so that the line A B shall be divided at C in *extreme and mean ratio* (**20**). From A as a centre, and with A B as radius, describe a circle; and from B as a centre, with A C for radius, draw an arc cutting the circle in the point D. Join B D and A D; then B A D is the triangle required, having B D equal to A C, the given length of the base, and each angle at B and D double the angle at A.

26. *To construct an equilateral-triangle upon a given finite right line.*—Let A B (fig. 22) be the given line. From A and B as centres, and with A B for radius, describe circles intersecting in the point C. Join A C, B C; and A C B is an equilateral triangle, having A C and B C each equal to A B.

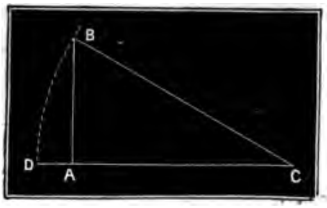
Fig. 22.



The angles are also equal, being each of them two-thirds of a right angle; hence in laying out an equilateral triangle on a large scale it is best to construct the two angles C A B and C B A, each equal to two-thirds of a right angle (60°), and the point C will be found.

27. *To construct a right-angled-triangle of which two sides are given.*—First, let the given sides be those which *include* the right angle.

Fig. 23.



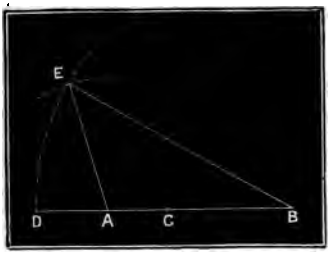
Draw two lines at right angles to each other (**3**), as A B, A C (fig. 23); and make A C and A B equal to the given sides respectively. Join B C; then B A C is the triangle required, having the right angle at A. The side B C which subtends or is opposite the right angle, is termed the hypotenuse, and is always greater than either of the other sides.

Secondly, let the base and hypotenuse be given. Draw any two lines, as A C and A B, at right angles to one another, and make A C equal to the given base. Take C D, equal to the given hypotenuse, and from C as a centre, and with C D for radius, describe a circle cutting A B in the point B. Join B C, and C A B is the triangle required.

28. *To construct a triangle whose three sides are of given lengths.*—(In this case the *sum* of any two of the given lengths must be greater than the third.)

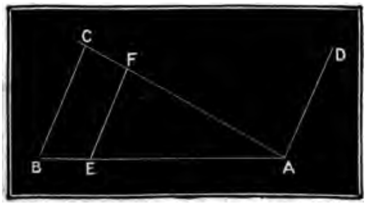
Draw a right line, B D (fig. 24), and take A B, equal to one of the given sides, B D equal to another, and D C to the third. From B as a centre, with B D as radius, describe a circle; and from A as a centre, with D C for radius, draw an arc cutting the circle in the point E. Join E A, E B; and B A E is the triangle required.

Fig. 24.



29. *To construct a triangle of which two sides and the included angle are given.*—Let C A B (fig. 25) be the given angle. Take A B and A C, equal to the given sides respectively, and join B C. Then B A C is the triangle required.

Fig. 25.



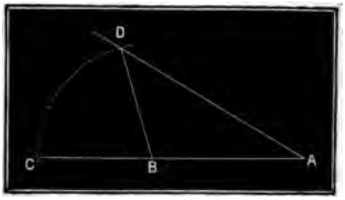
30. *To construct a triangle of which two angles and a side are given.*—(The sum of the given angles must be less than two right angles.) First, let the base, as A B (fig. 25), be the given side, and C A B, C B A the given angles adjacent to the base. Construct the angles C A B, C B A, respectively equal to the given angles (**7**); then the lines B C and A C will meet at C, and the triangle is completed.

Secondly, let the base A B (fig. 25) and the angles C A B, B C A be the given angles, one of which B C A is opposite the base or given side. Construct the angle C A B equal to one given angle, and the angle C A D equal to the other (**7**). Draw B C parallel to A D (**13**), meeting A C in the point C; then the angle A C B is equal (**12**) to the angle C A D, and the triangle B A C is the one required.

Another method is to take any point F on A C and draw F E, making the angle E F A equal to the given angle, and then drawing B C parallel to E F.

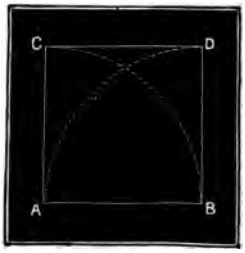
31. *To construct a triangle of which two sides and an angle not included are given.*—Draw a right line as A C (fig. 26), and take A B equal to one of the given sides, B C equal to the other. At A make the angle B A D equal to the given angle (**7**), and from B as a centre, with B C for radius, describe a circle cutting A D in the point D. Join B D, and the triangle B A D is the one required, having B A and B D respectively equal to the given sides, and B A D to the given angle.

Fig. 26.



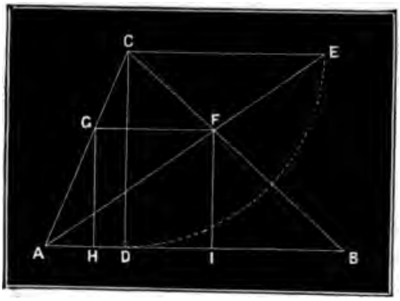
32. *To construct a square upon a given right line.*—Let A B (fig. 27) be the given line. Erect the perpendiculars A C, B D (**3**), and make A C and B D each equal to A B. Join C D; then the figure A B D C is the square required, having each side equal to A B.

Fig. 27.



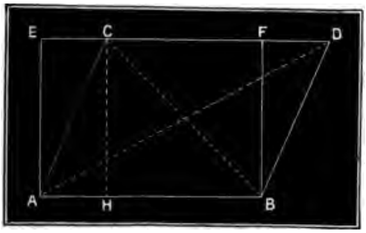
33. *In a given triangle to inscribe a square.*—Let A B C (fig. 28) be the given triangle. Draw C D perpendicular (**4**) to A B, and draw C E parallel to A B, making C E equal to C D. Join A E by a right line cutting B C at F. Draw F I parallel to C D, and F G parallel to A B and cutting A C at G; draw G H parallel to C D. Then F G H I is a square inscribed in the given triangle.

Fig. 28.



34. *To construct a parallelogram of given sides and given angle.*—Draw a right line A B (fig. 29) and construct the angle C A B equal to the given angle (**7**). Make A B and A C equal respectively to the given sides, draw C D parallel to A B, B D parallel to A C (**13**); then the figure A C D B is the parallelogram required.

Fig. 29.

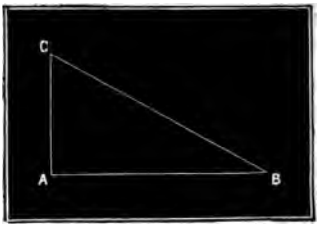


By drawing the diagonals A D and B C, the figure is divided into four triangles C A B, A B D, B D C, D C A, each of which is equal in area to one-half that of the parallelogram itself.

If the lines A E, B F are drawn perpendicular to A B and meeting C D in E and F, a rectangle A B F E is constructed which is equal in area to the first drawn parallelogram, and to double that of either of the triangles A C B or A D B. The perpendicular C H dropt upon A B is equal to A E, and represents the height of the vertex of the triangle A C B above the base A B; hence, when a triangle and a parallelogram are upon the same base and have the same vertical height, the area of the parallelogram is double that of the triangle.

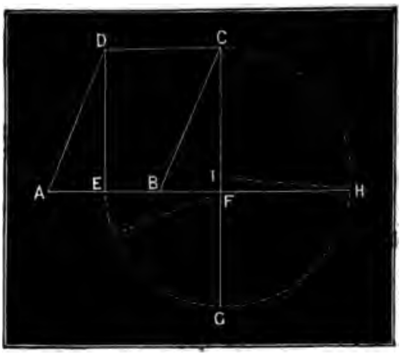
35. *To construct a square equal in area to the sum of the areas of two given squares.*—Let A B and A C (fig. 30) be two lines drawn at right angles. Take A B and A C equal in length to the sides of the given squares respectively; join C B; then B C is the side of the square which equals in area the sum of the squares upon A B and A C.

Fig. 30.



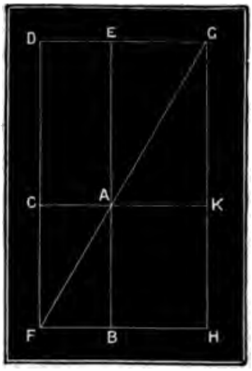
36. *To construct a square which shall be equal in area to a given parallelogram.*—Let A B C D (fig. 31) be the given parallelogram. Produce one side A B to H, and draw D E and C F at right angles to A H (**4**). Then the rectangle C D E F is equal in area to the given parallelogram (**34**). Produce C F to G, making F G equal to F E; bisect C G in I. From I as a centre and with I G as radius, describe a circle cutting A H in the point H. Then the square constructed on the line F H will equal in area the rectangle C D E F, and therefore will be equal to the given parallelogram.

Fig. 31.



37. *Upon a given right line to construct a rectangle equal* *in area to a given rectangle.*—Let A C D E (fig. 32) be the given rectangle. Produce E A to B, making A B equal to the given line. Through B draw F B H parallel to C A (**13**), and produce D C to meet F H in F. Draw the diagonal F A and produce it to meet D E in G. Draw G K H parallel to E B, meeting F H in H. Produce C A to meet G H in K. Then the figure A B H K is the required rectangle equal in area to the given one and drawn upon the given line.

Fig. 32.



38. *To construct a rectangle equal in area to a given irregular rectilinear figure.* — Let A B C D E (fig. 33), be the given figure. Draw the lines B E, E C so as to divide the figure into triangles. Draw A F perpendicular to B E (**4**), B G and D H to E C. Construct (**34**) the rectangle P (fig. 34) having the side I K equal to A F, and the side K L equal to half the line E B; then the area of P is equal to that of the triangle E A B. In the same manner construct a rectangle Q equal to the triangle E B C, and draw upon K L (**37**) a rectangle K L N M equal in area to Q. Draw a rectangle R equal to the triangle E D C and construct upon M N a rectangle

Fig. 33.

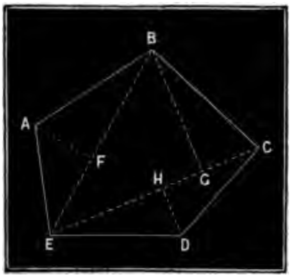
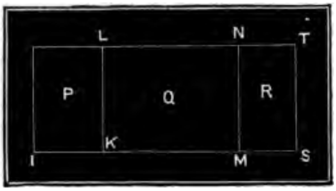


Fig. 34.



equal to R. Then the whole rectangle I T is equal in area to the given figure.

39. *To construct a rectilinear figure of a given irregular form.*—Let A B C D E (fig. 33) be the given figure, as for example, the plan of a field. In order to lay this down on paper we must not only measure the sides A B, B C, &c., but also the diagonals E B, E C. First draw one of the sides as A B, then from A as a centre, with A E as radius, describe a circle; and from B as a centre, with B E as radius, describe an arc cutting the circle in E; then the positions of two sides A B and A E are fixed. Next, from E as a centre, with E C as radius, describe a circle; and from B as a centre, with B C as radius, draw an arc cutting the last circle in C: join B C. From C as a centre, with C D as radius, describe a circle; and from E as a centre, with E D as radius, draw an arc cutting the last circle in D; join C D, E D, and the figure is completed.

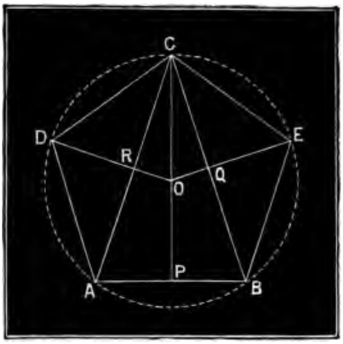
In this manner the outline of any irregular figure can be laid down by dividing it into as many triangles, less two, as the figure has sides.

40. *To divide a given finite right line into two parts, so that the rectangle under the whole line and one of the parts shall be equal in area to the square upon the other part.*—Let A B (fig. 17) be the given line. Draw E A D at right angles to A B (**3**). Bisect A B in C and make A E equal to AC; join E B, and make E A D equal to E B. Cut off A F equal to A D, and the rectangle under A B and B F is equal to the square on A F.

41. *To produce a given right line so that the rectangle under the whole produced line and the part produced shall* *be equal in area to the square upon the given line.*—Let A B (fig. 18.) be the given line. Bisect A B in C and draw C D at right angles to A B (**3**), making C D equal to A B. Join A D, and produce A B to E, making C E equal to A D. Then the rectangle under A E and E B is equal in area to the square upon A B.

42. *To construct a regular pentagon, or figure of five sides, the length of a side being given.*—Let A B (fig. 35) be the given side. On A B construct (**25**) an isosceles triangle A C B in which each angle C A B and C B A is double the angle A C B. Bisect the sides of the triangle in the points P, Q, R; and draw the lines P O, Q O, R O, at right angles to the sides respectively. These perpendiculars will all meet in one point O. Join O C, and from O as a centre, with O C for radius, describe a circle, which will pass through each vertex of the triangle A B C. Produce O Q and O R to meet the circle in E and D. Join C D, C E, E B, A D; then the figure A B E C D is the pentagon required, each side being equal to A B.

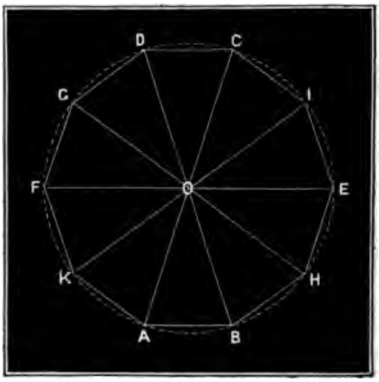
Fig. 35.



The ratio of A B to O P is very nearly as 1 : ·688, and that of A B to B C is very nearly as 1 : 1·618.

43. *To construct a regular decagon, or figure of ten sides, the length of a side being given.*—Let A B (fig. 36) be the given side. Construct (**25**) upon A B an isosceles triangle A O B having each angle O A B and O B A double the angle A O B. From O as a centre, with O A as radius, describe a circle. Produce A O and B O to meet the circle in C and D, and bisect the angles D O A and C O B (**8**) by the diameter E O F. Bisect each angle C O E, E O B, A O F, F O D, by the diameters I O K and H O G. Join all the points where these diameters meet the circumference by the lines C I, I E, E H, H B, &c., and the required decagon is completed, having each side equal to A B.

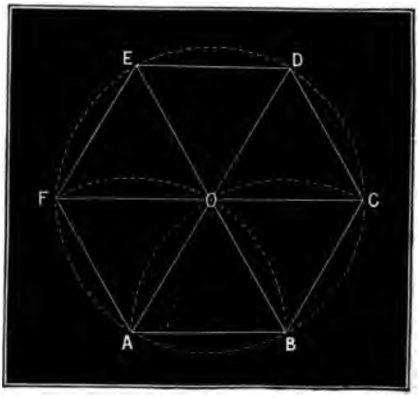
Fig. 36.



The ratio of A B to B O is very nearly as 1 : 1·618.

44. *To construct a regular hexagon, or figure of six sides, the length of a side being given.*—Let A B (fig. 37) be the given side. On A B (**26**) construct an equilateral triangle A O B; and from O as a centre, with O A as radius, describe a circle. Produce A O and B O to meet the circle in D and E; and from A and B as centres, with A O as radius, draw arcs cutting the circle in C and F. Join B C, C D, D E, E F, F A, and the hexagon is completed, each of these lines being equal to A B.

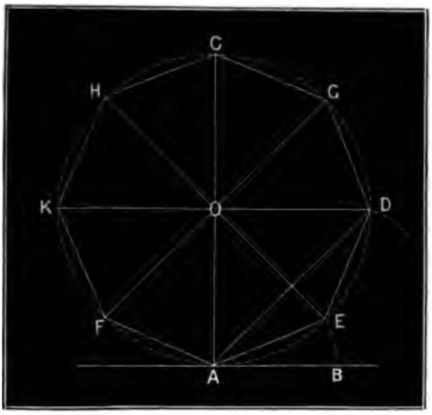
Fig. 37.



It is evident that A O is equal to A B.

45. *To construct a regular octagon, or figure of eight sides, the length of a side being given.*—Let A B (fig. 38) be the length of the given side. Draw A C at right angles to A B, and bisect the angle B A C by the line A D (**8**). Bisect the angle D A B by the line A E, and from A as a centre, with A B as a radius, draw a circle cutting A E in the point E. From E as a centre, with E A for radius, describe a circle cutting A D in the point D. Join E D and A D. Draw D O K parallel to A B intersecting A C in the point O. Through O draw F O G parallel to A D. From O as a centre, with O A for radius, describe a circle passing through A, E and D. Draw the diameter E O H and join all the points where the diameters meet the circle by the lines D G, G C, C H, &c., and the required octagon is completed, each side being equal to A B.

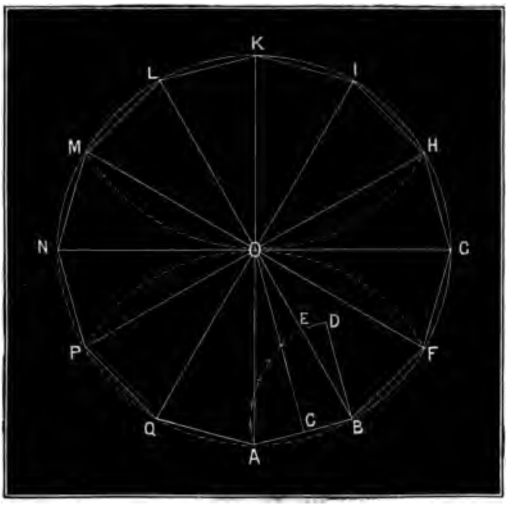
Fig. 38.



The ratio of A B to A O is nearly as 1 : 1·307.

46. *To construct a dodecagon, or figure of twelve sides, the length of a side being given.*—Let A B (fig. 39) be the given side. Erect the line B D (**3**) at right angles to A B, and from B as a centre, with B A for radius, describe a circle cutting B D in the point D. Divide the right angle D B A into twelve equal parts ( ), and take D B E equal to two of those parts. Bisect A B in C, and draw the line C O perpendicular to A B meeting B E in O. From O as a centre, with O A or O B as radius, describe a circle through B and A. Produce A O and B O to meet the circle in K and L. From A and K as centres, with O A for radius, describe arcs cutting the circle in P, F, H and M. Draw G O N at right angles to A K, and bisect the angle A O P by the line Q O I. Join all the points B, F, G, H, &c., upon the circle, and the dodecagon will be completed, having each side equal to A B.

Fig. 39.



The ratio of A B to A O is nearly as 1 : 1·932.

In the same manner we can construct a regular polygon of any given number of sides, provided that given number is some product of 3 into a power of 2. The right angle A B D must be divided (**9**) into the same number of parts as there are to be sides to the figure, and the angle D B E taken equal to two of those parts; then the point O where B E meets the bisecting perpendicular C O will be the centre of the circumscribing circle. Then from A and B as centres, with A B for radius, describe arcs cutting the circle in

Q and F, and so proceed round the circle until all the points where the polygon touches the circle are found.

In this way a quindecagon or fifteen-sided figure can be constructed; for (**10**) we can divide the right angle A B D (fig. 39) into fifteen equal parts. Take D B E equal to two of those parts and proceed in the construction the same way as for the dodecagon.

For a figure of sixteen equal sides, we must divide the right angle A B D into eight equal parts (**8**) and take D B E equal to one of them.

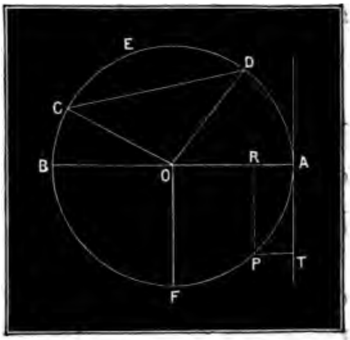
For a figure of twenty equal parts we must first divide the angle A B D into five equal parts (**10**) and bisect each of those parts, by which the angle will be divided into ten equal parts; and D BE being taken equal to one of those parts the figure can be constructed in the manner above described.

CHAPTER II.

THE CIRCLE.

47. Definitions.—A *curve* is a line made by the motion of a point the direction of which is constantly changing. A *circle* is a curve produced by the motion of a point about a fixed point from which it is always at the same distance; the fixed point about which the motion takes place is called the *centre* of the circle; and the curved line thus produced is called the *circumference*. If a straight line or rod O A (fig. 40) is made to revolve about one of its extremities O, its other extremity A will trace out a circle, and the length O A is called the *radius* of circle; the line A O B, which is double the radius, is called the *diameter* of the circle. If a line A T is drawn at right angles (**3**) to A O, at the outer end of any radius, A T is called the *tangent* to the circle, which it touches without cutting. Any line as C D which is drawn across the circle but does not pass through the centre is called a *chord*. The part of the circle C E D cut off by the chord is called an *arc*, and the space included between the arc and chord is called a *segment*. If we draw the lines O C, O D, from the centre to the ends of the

Fig. 40.



chord the space included between those lines and the arc is termed a *sector*. The diameter A O B divides the circle into two equal parts, each of which is called a *semicircle*. The perpendicular O F upon A B divides the semicircle into two equal parts A F, B F, called *quadrants*. Circles are said to touch each other when their circumferences meet at a point but do not cut one another, and the right line joining their centres passes through the point of contact. Two or more circles are said to be *inscribed* in another circle when they touch its circumference on the inner or *concave* side, and also touch each other on the outer or *convex* side. Two circles drawn from the same centre but with different radii are said to be *concentric*.

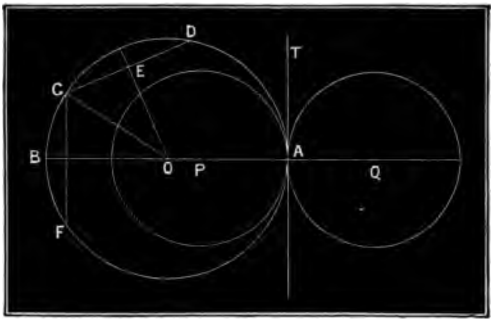
A rectilinear figure is *inscribed* in a circle when all its angular points, or vertices, are on the circumference of the circle, and the circle is then said to be *circumscribed* about the rectilinear figure. A rectilinear figure is *circumscribed* about a circle when all its sides are tangents thereto, and the circle is then said to be *inscribed* in the figure.

48. *To draw a tangent at a given point of a given circle.*—Let O (fig. 40) be the centre of the circle, A the given point on its circumference; join A O. At the point A draw A T perpendicular to A O; then A T is the tangent required.

49. *To describe a circle of given radius touching a given circle in a given point.*—Let O (fig. 41) be the centre of the given circle, A the point on its circumference where the contact is to take place. Draw the radius O A and produce it to Q, making A Q equal to the given radius. From Q as a centre, with Q A as radius, describe a circle which will touch the given one at the point A as required.

If the given radius is less than that of the given circle, cut off from A O the length A P equal to the given radius, and from P as a centre, with P A as radius,

Fig. 41.



describe a circle whose convex side will touch the given one on the concave side of its circumference at the point A.

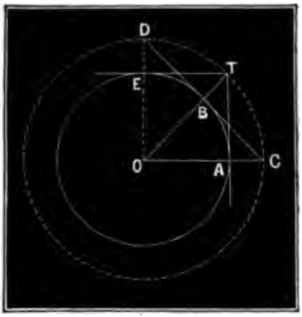
If P is the centre of the given circle, and the given radius is greater than P A, produce A P to O, making A O equal to the given radius. Then a circle drawn from O as a centre, with O A as radius, will touch the given one on the convex side of its circumference.

If A T is drawn perpendicular to O A Q it will be the common tangent of the two circles which are in contact at A.

50. *To describe a circle through three given points which are not in the same right line.*—Let D, C and F (fig. 41) be the three given points. Join C D and C F, and bisect C D and C F at E and B. Draw E O at right angles to C D, B O at right angles to C F; these will meet at the point O which is the centre of the required circle. Join O C; then a circle described from O as a centre, with O C as radius, will pass through the three given points.

51. *From a given point outside a given circle to draw* *tangents thereto.*—Let O (fig. 42) be the centre of the given circle, O A its radius, T the given point outside the circle. Join O T; then from O as a centre, with O T as a radius, describe another circle concentric with the given one. Let B be the point where O T cuts the given circle, draw D B C at right angles to O B, and cutting the outer circle in D and C. Draw the lines O D, O C, cutting the given circle in E and A; join T E, T A; then T E and T A are the required tangents at the points E and A of the given circle.

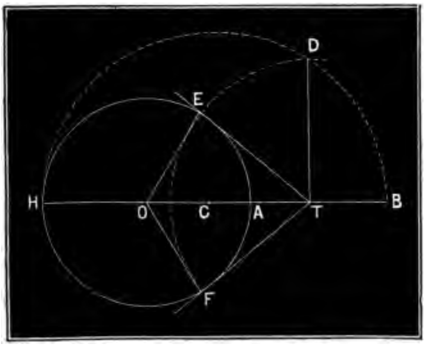
Fig. 42.



Another method.

Let O (fig. 43) be the centre of the given circle, T the given point. Draw the diameter H O A and produce it to T and B; making T B equal to T A. Bisect H B in C, and from C as a centre with C B as radius, describe a semicircle. Draw T D at right angles to H B, meeting the semicircle in D: from T as a centre, and with T D for radius, draw an arc, cutting the given circle in E and F. Join O E, O F, T E, T F; then T E and T F are the tangents required.

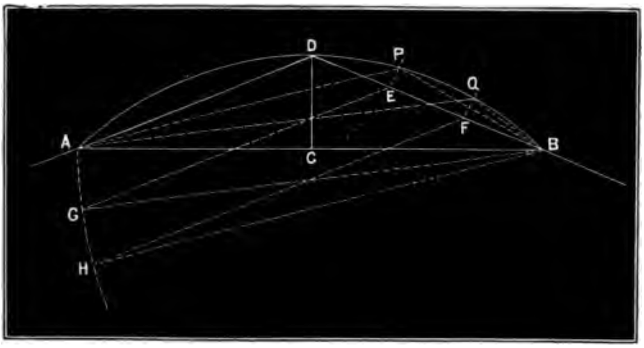
Fig. 43.



This solution depends on the property that the rectangle under H T and T A is equal to the square on the tangent T E or T F, and also that the rectangle under H T and T B is equal to the square on T D (**36**).

52. *To find any number of points upon an arc of a circle, the middle point and two extremities of the arc being given.*—Let A and B (fig. 44) be the two extremities of the arc, D the middle point. Join A B; and bisect the chord A B in C; draw D C which will be perpendicular to A B. Draw the chords A D, D B, and divide D B into any number of parts by the points E, F, &c. From B as a centre, and with B A as radius, describe a circle A G H. Draw E G, F H parallel to D A, cutting the circle in G and H. From B as a centre, and with radius B E, describe a circle; and from A as a centre, with G E as radius, draw an arc cutting the last circle in P. Then P is a point on the required arc. In the same manner from B as a centre, and with radius B F, describe a circle; and from A as a centre, with H F as radius, draw an arc cutting this circle in Q; then Q is also a point on the required arc. In this way any number of points can be found upon the arc of a circle when the centre is at too great a distance to be available; the arc itself can either be drawn by hand or by help of a bent lath passing through all the points.

Fig. 44.



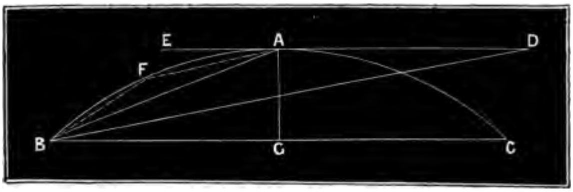
This method is derived from the principle that all the angles in the same segment of a circle are equal; for the angles A D B, A P B, A Q B, are all equal. The *entasis* or curved outline of the shaft of a column or the contour of a spire can be drawn on a large scale in this way.

53. *To draw an arc of a circle by continuous motion, when the centre is not available.*—Take two straight rulers of twice the length of the chords A D, B D (fig. 44), and fasten them together so as to make with each other an angle equal to A D B, their outer edges being applied to the chords A D, D B. Fix a pin at A and another at B against the outer edges of the rulers; let a pencil be fixed at the vertex D, and as the rulers move backwards or forwards against the pins the pencil at D will mark out the arc of the circle.

Another method.

Let B C (fig. 45) be the chord of the arc, A G its height at the centre, E A D the tangent at A perpendicular to A G, and parallel to B C. Draw the chord A B, and take A D equal to A B; join B D. Now let the triangle B A D be made rigid, and let it move about pins fixed at A and B; then a pencil fixed at its vertex A will mark out the arc A F B as the triangle moves from right to left. So also by making the triangle move from left to right we can trace the arc A C.

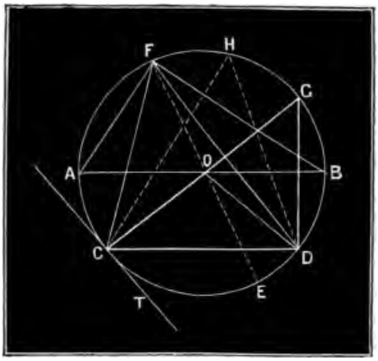
Fig. 45.



This method is based on the principle that if we take any point F in the arc A B, and draw the chords A F, B F, then the angle B F A equals the angle B A D.

54. *On a given right-line to describe a segment of a circle that shall contain a given angle.*—Let C D (fig. 46) be the given line, making with C T the angle D C T equal to the given angle. Draw C O G perpendicular to C T, and draw D O, making with D C an angle O D C equal to the angle O C D. From O as a centre, and with O C or O D as radius, describe a circle cutting C O G in the point G; join G D. Then the angle C G D is equal to the given angle D C T.

Fig. 46.



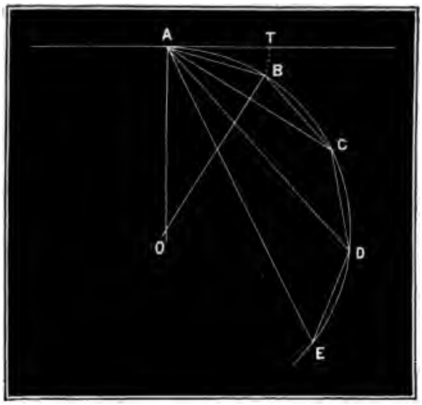
If we take any other points as F, H, on the circumference and join them with C and D, the angles C F D, C H D will be each equal to the angle C G D, or to the given angle D C T. The angle C O D at the centre of the circle is double the angle C G D, and double the angle D C T.

If A B is the given line, and the given angle is a right angle, bisect A B in O, and from O as a centre, and with O A as radius, describe a circle. Take any point F on the circle and draw the chords F A, F B; then the angle A F B is a right angle.

55. *From the extremities of a given right-line to draw two other lines in any directions meeting at right angles to each other.*—Let A B (fig. 46) be the given line, and let B F be the given length of one of the other lines less than A B. Bisect A B in O, and from O as a centre describe a semicircle on A B. From B as a centre, and with B F as radius, draw an arc cutting the circle in F; join A F: then A F B is a right angle.

56. *To lay out an arc of a circle as a railway curve of given radius, by finding points on the circumference.*—Let A T (fig. 47) be the tangent to the proposed curve at the point A, O the centre of the circle. Suppose it is required to find points on the arc at distances apart equal to a given length A T. From A as a centre, and with A T as radius, draw an arc T B, and construct the angle T A B equal to half the angle A O B at the centre of the circle. Then B is the first point on the curve. Make the angles B A C, C A D, D A E, each equal to B A T; and from B as a centre, with B A as radius, draw an arc cutting A C in C; from C as a centre, and with C B as radius, draw an arc cutting A D in D; from D as a centre, and with D C as radius, draw an arc cutting A E in E; then A, B, C, D, and E, are all upon the required curve.

Fig. 47.



This method is based on the principle that angles which stand upon equal arcs of a circle and have their vertices on the circumference are all equal.

57. *To find arithmetically the radius of an arc of a circle of which the chord and height are given.*—Let B C (fig. 45) be the given chord whose length is known arithmetically, A G the given height. Bisect B C in G. Add together the squares of the lengths A G and B G, and divide the sum by twice A G, the quotient is the radius of the circle. For example, let B C equal 10, A G equal 2;

$5 \times 5 + 2 \times 2 = 29$, $2 \times 2 = 4$, 29 divided by 4 is $7\frac{1}{4}$ the length of the radius.

58. *To describe a circle touching a given right-line and passing through two given points.*—First, let the line joining the two points be not parallel to the given line. Let E C (fig. 48) be the given line, P and Q the given points. Join P and Q, and produce P Q to meet the given line in C. Produce P C to G, making G C equal to C Q. On P G describe a semicircle, and draw C F perpendicular to P C, meeting the circle in F. Take C E equal to C F, and draw E X perpendicular to C E. Bisect P Q in D, and draw D X perpendicular to P Q, and meeting E X in the point X. Then the circle described from X as a centre, and with X E for radius, will touch the given line at E and pass through the points P and Q.

Fig. 48.

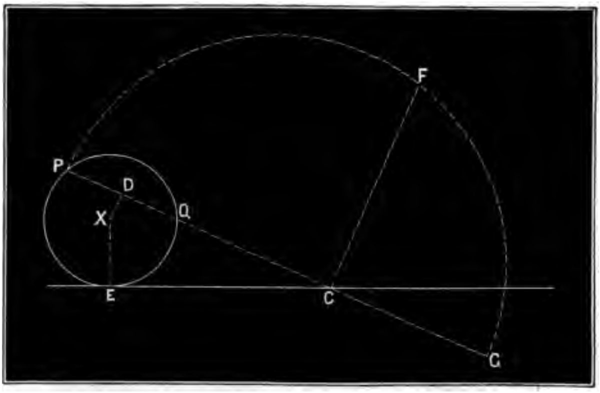
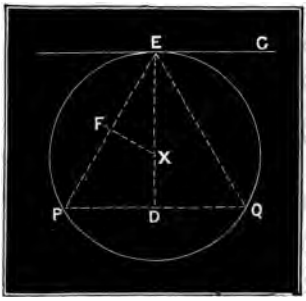


Fig. 49.

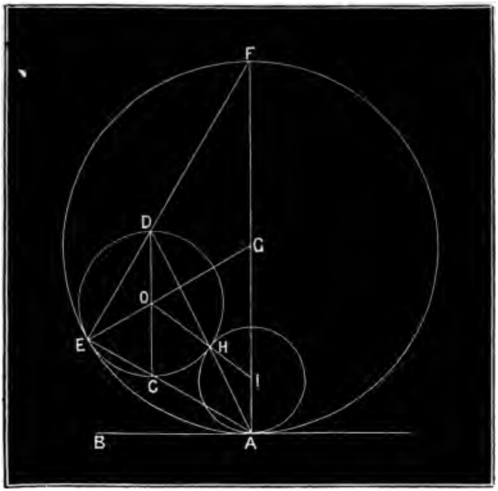


Secondly, let the line joining the points be parallel to the given line. Let E C (fig. 49) be the given line, P and Q the given points. Join P Q, and bisect

P Q in D. Draw D E perpendicular to P Q, and join E P. Bisect E P in F, and draw F X perpendicular to E P, meeting D E in X. Then the circle described from X as a centre, and with X E for radius, will touch the given line at E and pass through the given points P and Q.

59. *To describe a circle touching a given circle, and also a given right-line outside of it in a given point.*—Let O (fig. 50) be the centre of the given circle, A B the given line wholly outside of the circle, A the given point therein. Draw A F perpendicular to the line A B, and also a diameter C O D parallel to A F. Join D A and C A, producing A C to meet the circle again at E. Join E O, and produce E O to meet A F in G; join E D, producing it to meet A F in F. Then G F is equal to G A or to G E; and from G as a centre, with G A as radius, draw a circle which will touch the given circle in E, and the given line at the given point A.

Fig. 50.

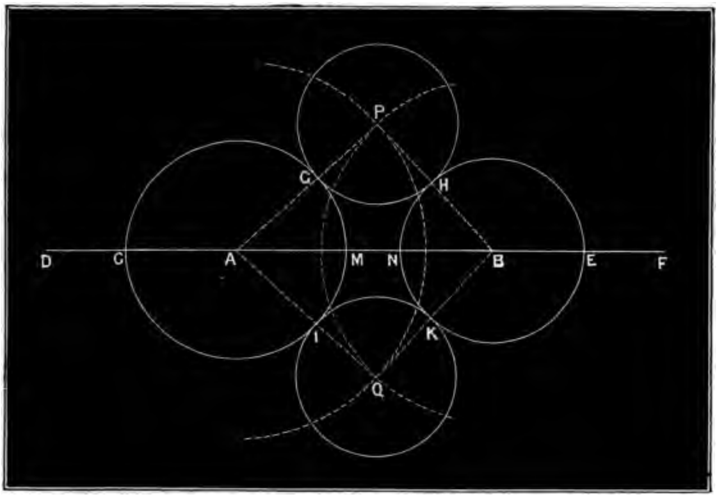


Again, let A D cut the given circle at H; join O H, and produce O H to meet A F in I. Then I H is equal to I A; from I as a centre, and with I A as radius, describe a circle which will touch the given circle at H and the given line at A.

60. *To describe a circle of given radius which shall touch two given circles.*—First, suppose the given circles do not intersect, their distance apart being not greater than the diameter of the required circle.

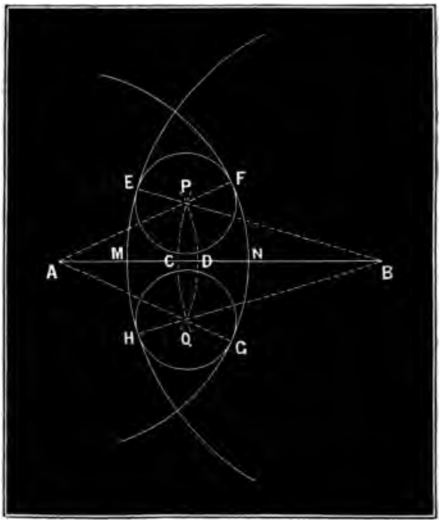
Let A, B (fig. 51) be the centres of the given circles, joined by the indefinite right line D F, which cuts the circumferences in C, M, N, and E. Take C D, E F, each equal to the given radius; from A as a centre, and with A D as radius, describe a circle; and from B as a centre, with B F as radius, draw arcs cutting the last circle in P and Q. Join the points A, P, B, Q; then a circle drawn from either P or Q as centre, and with the given length of radius, will touch the given circles in G and H or in K and I.

Fig. 51.



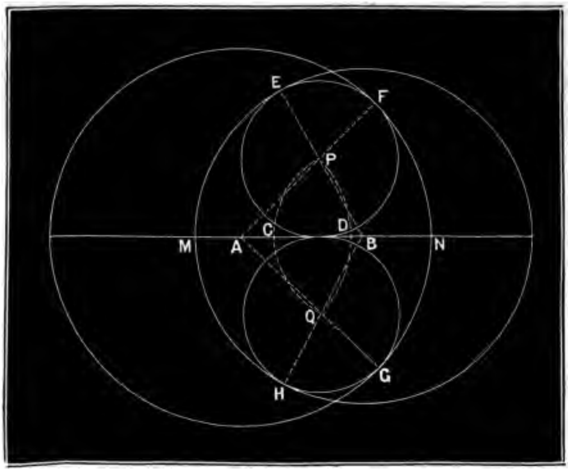
Secondly, suppose the given circles intersect each other, and the required circle is to touch their concave sides, its given radius being not greater than half the distance apart of their circumferences as measured on the line joining the centres.

Fig. 52.



Let A, B (figs. 52, 53, 54) be the centres of the circles, joined by the line M N, meeting the circumferences in M and N. Take M C, N D each equal to the length of the given radius; from A as a centre, with radius A D, describe a circle, and from B as a centre, with radius B C, draw arcs cutting the last circle in P and Q.

Fig. 53.



Join the points A, P, B, Q, producing A P, A Q, B P, B Q, to meet the circles in F, G, H, and E. Then a circle drawn from either P or Q as centre and with the given length of radius will touch the given circles in E and F or in G and H; P F, P E, Q H, Q G, being each equal to the given radius M C or N D.

Fig. 54.

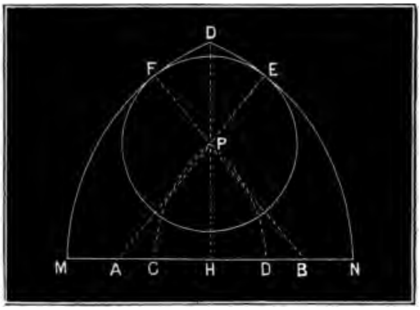
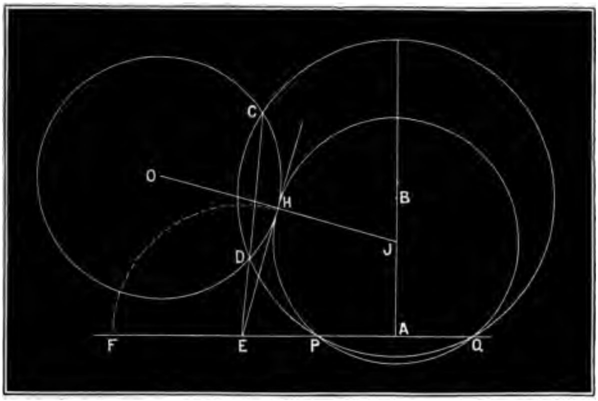


Fig. 54 represents a Gothic arch struck from centres A, B, in which the circle of radius equal to M C or N D is inscribed.

61. *To describe a circle through two given points and touching a given circle.*—First, let the given points be outside the given circle. Let O (fig. 55) be the centre of

Fig. 55.

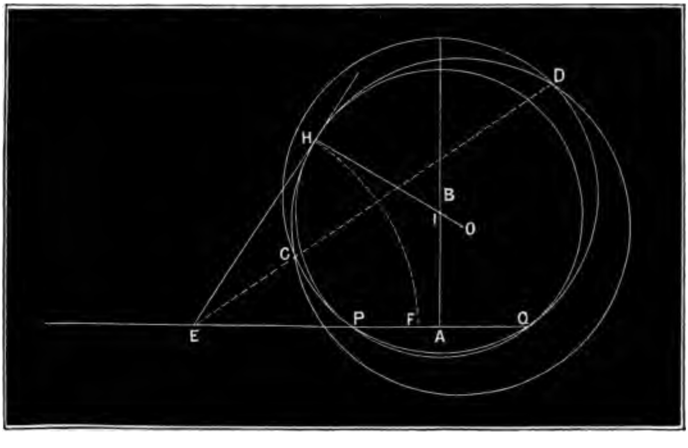


the given circle, P and Q the given points. Through P and Q draw an indefinite right-line F P Q; bisect P Q in A, and draw A B perpendicular thereto. Take any point B, on the perpendicular A B, as a centre, and describe a circle that will cut the given circle in any two points C, D, and also pass through P and Q. Draw the chord C D, and produce it to meet the line through F P Q, in E. Find E F a mean proportional (**17**) between E P and E Q, and from E as a centre, with E F as radius, describe a circle cutting the given circle in H; join O H, and produce O H to meet A B in J. Then a circle drawn from J as a centre, with J H as radius, will touch the given circle and also pass through the given points.

Secondly, let the given points be within the circumference of the given circle.

Let O (fig. 56) be the centre of the given circle, P, Q, the given points. Draw through P and Q an indefinite right-line; and bisect P Q in A. Draw A B perpendicular to P Q, and from any point B therein describe a circle that shall cut the given circle in any points C and D, and also pass through the given points P and Q. Join C D, and produce D C to meet P Q in E. Find E F a mean proportional (**17**) between E P and E Q, and from E as a

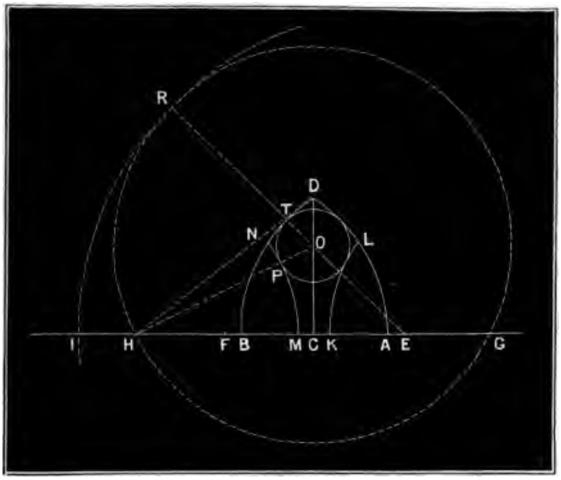
Fig. 56.



centre, with E F as radius, describe a circle cutting the given one in H; draw H O cutting A B in the point I; then a circle drawn from I as a centre, and with I H as radius, will touch the given one and also pass through the given points.

62. *To draw geometrical tracery in a Gothic arch.*—Let B D A (fig. 57) be a Gothic arch springing from the line A B, joining the centres E, F, from which the arcs B D, A D, are struck. Bisect A B in C, and draw C D perpendicular thereto. Take B M, A K equal to each other and less than half A B; make M H, K G each equal to A F or B E. From G and H as centres, and with G K, H M as radius, describe the arcs M N, K L. It is required to draw a circle which shall touch all four of the arcs, and have its centre on C D.

Fig. 57.



Take B I equal to B E, and from E as a centre, with E I as radius, describe a circle; draw a circle (**61**) passing through G and H, and touching the last-drawn circle in the point R; this circle will have its centre at O on the line C D. Draw the lines R O, H O, cutting the given arcs in T and P; and from O as a centre, with O P or O T as radius, draw a circle, which will touch all four of the given arcs as required.

Let the given arch A D B (fig. 58) be equilateral, A being the centre of the arc B D, and B that of the arc A D; and suppose the line A B divided into three equal parts at K and M. Let G and H be the centres of the arcs M N, K L, having their radius equal to A B. Take C the centre of A B, and draw the vertical line C D. Measure M X equal to one-sixth of M B or to one-third of M C, and from A as a centre, with A X as radius, describe a circle cutting C D in O. Draw H P O cutting K L in P, and join A O, producing A O to meet B D in T. Then a circle drawn from O as a centre, and with O T or O P for radius, will touch all the four arcs as required.

Fig. 58.

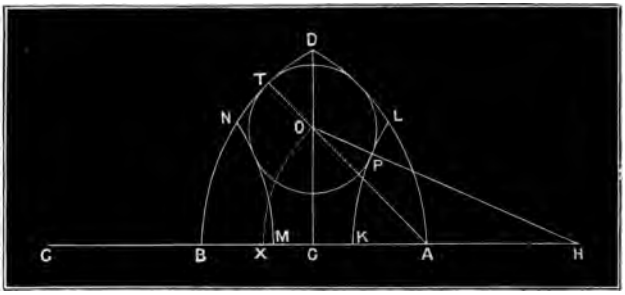
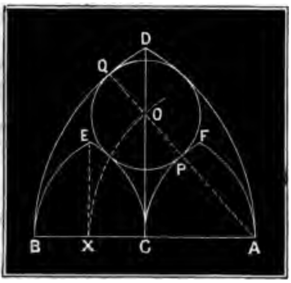


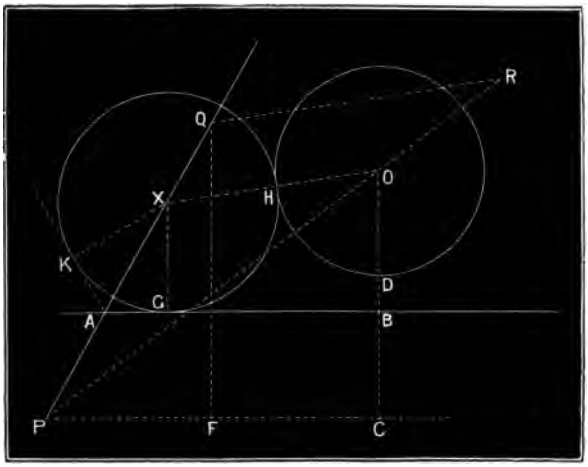
Fig. 59.



Let the given arch A D B (fig. 59) be equilateral, and also the inner arches A F C, B E C; A B being bisected in C. Draw the vertical C D; bisect B C in X, and from A as a centre, with A X as radius, describe a circle cutting C D in O. Draw the line A P O Q meeting C F in P and B D in Q. Then a circle described from O as a centre, and with O P or O Q as radius, will touch all four of the given arcs.

63. *To describe a circle which shall touch a given circle and a given right-line, and shall also have its centre on another given line.*—Let O (fig. 60) be the centre and O D

Fig. 60.



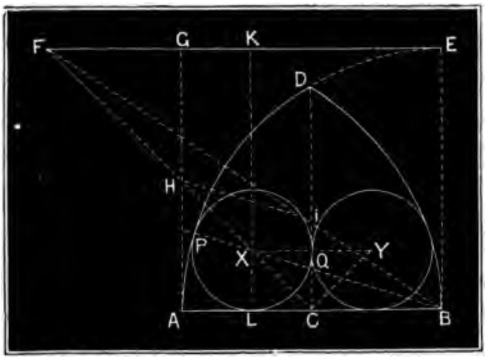
the radius of the given circle; A B the given line to be touched, P Q the given line on which the centre of the circle is to be found. Let P Q cut A B in the point A. From O draw O D B perpendicular to A B, and produce it to C, making B C equal to O D. Draw C P parallel to A B, meeting P Q in P; join P O, and produce P O indefinitely to R. Take any point Q on the line P Q, and drop therefrom the line Q F perpendicular to C P; from Q as a centre, and with Q F as radius, describe a circle cutting P R in R, and join Q R. Draw O H X parallel to R Q, meeting P Q in X, and draw X G perpendicular to A B. Then a circle described from X as a centre, and with X G or X H as radius, will touch A B at G and the given circle at H, as required.

64. *To describe a circle touching a given circle, and also two given right-lines which are not parallel.*—Let A K and A B (fig. 60) be the given lines meeting at the point A; O the centre and O D the radius of the given circle. Bisect the angle B A K by the line P A Q, then the centre of the required circle must lie upon this line. Find X the centre and X G the radius of the circle which touches the given circle and also the line A B at G (**63**); draw X K perpendicular to A K, and the circle drawn from X as a centre, with X G or X K or X H as radius, touches the two lines in G and K and the given circle in H.

INSCRIBED AND CIRCUMSCRIBED CIRCLES.

65. *To inscribe three circles in an equilateral Gothic arch.*—Let A D B (fig. 61) be the arch, having the arcs

Fig. 61.

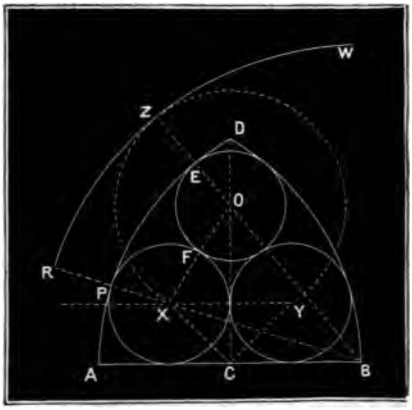


A D, B D struck from B and A as centres. Draw the horizontal springing line A B; it is required to inscribe three circles in the figure A B D.

Bisect A B at C and draw the perpendicular C D. Bisect the angles A C D, B C D by the lines C X, C Y (**8**). Draw A G, B E perpendicular and equal to A B; and draw E G F parallel to A B. Produce C X to meet G E in F, and cutting A G in H; join B F. From H as a centre, and with H G as radius, describe an arc cutting B F in I, and join H I. Draw B P parallel to H I and cutting C F in X; take C Y equal to C X and draw X Y, cutting C D in Q. Draw X L perpendicular to A B, and the circle drawn from X as a centre, with X P or X L or X Q as radius, will touch A B at L, C D at Q, and A D at P. Also a circle described from Y as a centre, and with the same radius, will touch C D at Q, and also the arc B D and the line B A.

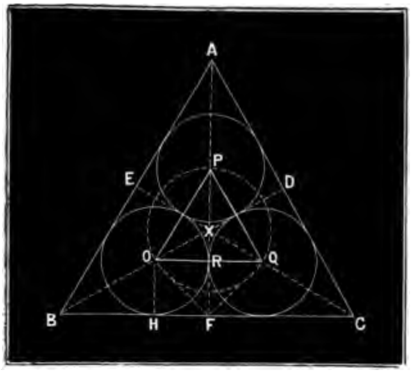
We have now to draw the circle which shall touch the circles X and Y and also the arcs A D, B D. Produce B P (fig. 62) to R making P R equal to P X; and from B as a centre, with B R as radius, describe a circle R Z W. Find O on C D (**61**) the centre of the circle which passes through X and Y and touches the circle R Z W in the point Z. Draw the lines X O, B O. Produce B O to meet A D in E, and let X O cut the circle X at the point F. Then a circle drawn from O as a centre, and with O E or O F as radius, will touch the arcs A D, B D, and also the two previously inscribed circles.

Fig. 62.



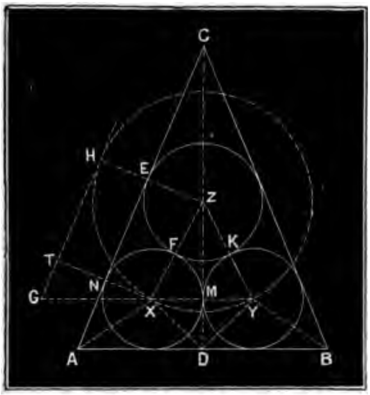
66. *To inscribe three circles in an equilateral triangle.*—Let A B C (fig. 63) be the triangle. Bisect each side at D, E, F; and join D B, E C, A F. The lines D B, E C, and A F will intersect in the point X, and the centres of the required circles will be found on these three lines. Bisect the angle A F B by the line F O meeting B X in O; and from X as a centre, with X O as radius, describe a circle cutting X A in P, X C in Q. Draw the lines O P, O Q, P Q; let O Q intersect A F in R, and the circles described from O, P, and Q as centres, with O R, or half O Q, for radius, will be inscribed in the triangle.

Fig. 63.



67. *To inscribe three circles in a given isosceles triangle.*—In this case two only of the circles will be equal. Let A C B (fig. 64) be the given triangle. Bisect the base A B at D and draw D C perpendicular thereto. Bisect each angle C A B, C D A, C D B, C B A, by the lines A X, D X, B Y, D Y. Draw X Y cutting D C in M; then the circles drawn from X and Y as centres, and with X M or Y M for radius, will touch each other and the sides of the triangle.

Fig. 64.

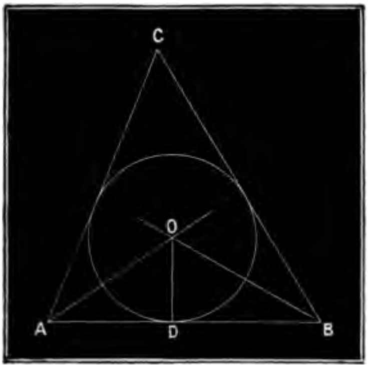


The centre of the third circle will lie in the line D C.

Draw X N perpendicular to A C and produce it to T, making N T equal to N X. Draw T H parallel to A C. Find on the line D C (**58**) the centre Z of the circle which touches T H at H, and also passes through the points X, Y. Draw Z H perpendicular to A C and cutting it at E. Join X Z, Y Z; and the circle drawn from Z as a centre, with Z E or Z F or Z K as radius, touches the sides A C, B C of the given triangle, and also the two previously inscribed circles at the points F and K.

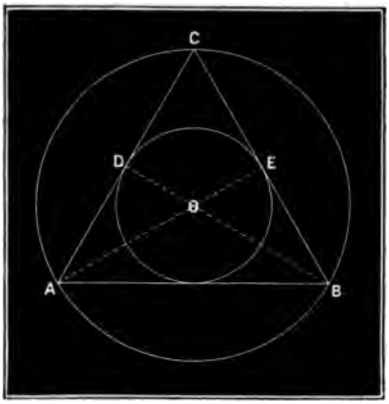
68. *In a given triangle to inscribe a circle.*—Let A B C (fig. 65) be the given triangle. Bisect the angles C A B, C B A by the lines A O, B O, meeting in the point O. Draw O D perpendicular to A B, and the circle drawn from O as a centre, with O D for radius, will touch each side of the triangle.

Fig. 65.



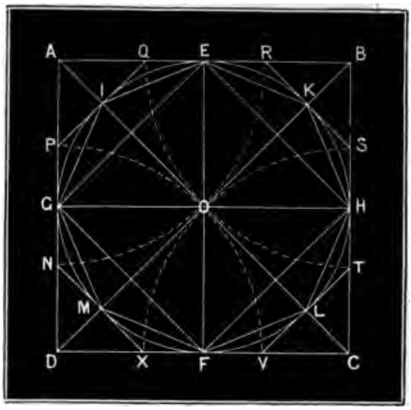
69. *In an equilateral triangle to inscribe a circle.*—Let A B C (fig. 66) be the given triangle. Bisect any two sides A C, B C by the points D and E. Draw B D, A E, intersecting at O. Then the circle described from O as a centre, and with O D or O E as radius, will touch the sides of the triangle. The point O will also be the centre of the circumscribing circle of radius O A.

Fig. 66.



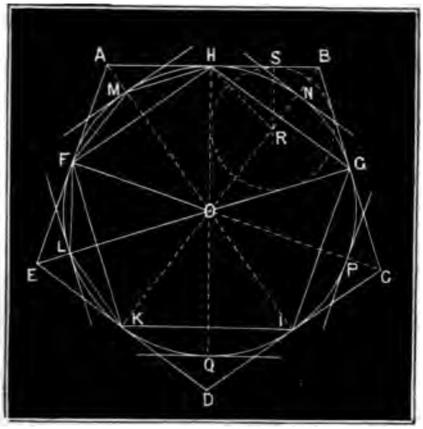
70. *In a given square to inscribe a circle.*—Let A B C D (fig. 67) be the given square. Draw the diagonals A C, B D, intersecting at O. Draw O E perpendicular to A B or parallel to A D, and the circle described from O as a centre, with O E as radius, will touch all the sides of the square.

Fig. 67.



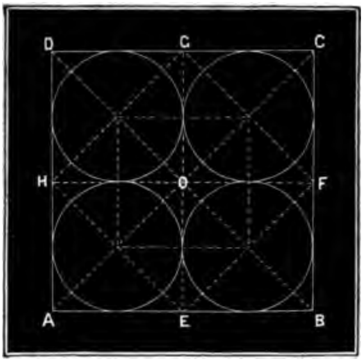
71. *In any regular polygon to inscribe a circle.*—Let A B C D E (fig. 68) be the given polygon. Bisect any two adjacent sides as A E, A B at F and H. Draw F O, H O at right angles to A E and A B; these will meet at a point O. Then the circle described from O as a centre, and with O F or O H as radius, will touch each side of the polygon.

Fig. 68.



72. *In a given square to inscribe four equal circles.*—Let A B C D (fig 69) be the given square. Draw the diagonals A C, B D, intersecting in O. Draw E O G parallel to B C, and F O H parallel to A B. In each of the four squares into which the figure is divided inscribe a circle (**70**), and the four circles will touch each other, and also the sides of the given square.

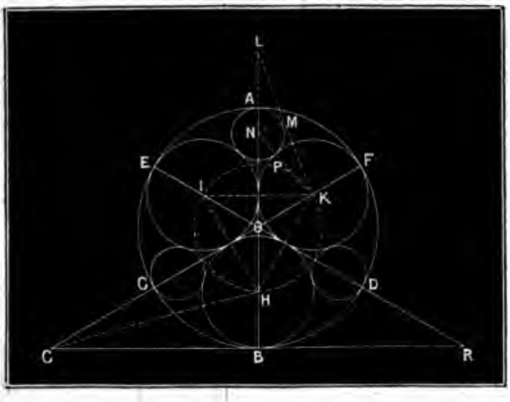
Fig. 69.



73. *In any given regular polygon to inscribe as many equal circles as the figure has sides.*—Let A B, B C (fig. 68) be two adjacent sides of the given polygon A B C D E. Bisect the sides in H and G; and draw H O, G O at right angles to A B, B C, and let them meet at O. Join O B and bisect the angle O H B by the line H R, meeting O B in R; draw R S perpendicular to H B. Then a circle described from R as a centre, with R S as radius, will touch the lines H B, B G, G O, and O H, and will be one of the inscribed circles. Proceeding in the same way at every corner of the figure, as many circles can be inscribed in it as the figure has sides or corners.

74. *In a given circle to inscribe three equal circles.*—Let O (fig. 70) be the centre, A O B one of the diameters of the given circle. From A and B as centres, and with O A as radius, describe arcs cutting the circle in C, D, E, F. Draw the tangent (**48**) G B R perpendicular to A B, and also the diameters F O C, E O D, producing them to meet the tangent at the points G and R. Bisect the angle O G B by the line G H cutting A B in H. From O as a centre, and with O H as radius, describe a circle cutting the other diameters in I and K; join H I, I K, K H; then circles drawn from H, I, and K as centres, with H B for radius, will touch each other, and also the given circle at the points B, E, and F.

Fig. 70.

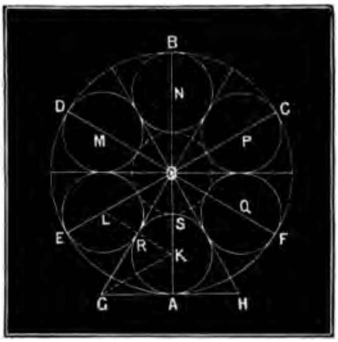


Having drawn the diameters A O B, E O D, F O C, we can find the centres of the circles by measurement on a scale; for if the radius O A of the given circle measures on any scale 1,000, then B H will be 464 and O H will be 536.

If it is required to draw three smaller circles touching the given one and the three inscribed circles, produce B A to L, making A L equal to H B. Draw L K and bisect it in M; let M N, the perpendicular to L K, cut the line A B in N; then a circle described from N as a centre, and with N A as radius, will touch two of the inscribed circles, and also the given circle at A. In the same way the other circles at C and D can be drawn with radii equal to A N.

75. *To inscribe six equal circles in a given circle.*—Let O be the centre, A O B (fig. 71) a diameter of the given circle. Draw the tangent (**48**) G A H, and also the diameters E O C, D O F, dividing the circle into six equal sectors as in the last problem (**74**). Bisect each of the angles E O A, A O F by the lines O G, O H, meeting the tangent in G and H. Bisect the angle O G H by G K, cutting A O in K. From O as a centre, with O K as radius, describe a circle cutting the diameters in the points, L, M, N, &c. Then circles drawn from the points K, L, M, &c., as centres, with K A as radius, will all touch the given circle and will touch each other. A circle drawn from O as a centre, and with radius O S equal to K A, will touch each of the inscribed circles. The radius of each of the inscribed circles in one-third that of the given circle.

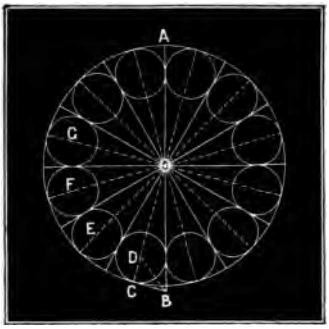
Fig. 71.



76. *To inscribe twelve equal circles in a given circle.*—

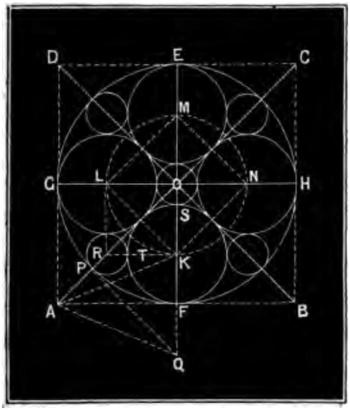
Let O (fig. 72) be the centre, A O the radius of the given circle. Divide the circle into twelve equal sectors as in the last problem (**75**), and bisect one of those sectors by the radius O C; draw the tangent (**48**) B C at C perpendicular to O C, and bisect the angle C B O by the line B D, cutting O C in D. Bisect each of the other angles, and, from O as a centre, with O D as radius, draw a circle cutting the radii in the points, E, F, G, &c. Then circles drawn from the points D, E, F, &c., as centres, and with D C as radius, will touch each other and the circumference of the given circle.

Fig. 72.



Taking the radius O A as 1,000 on any scale, the length of the radius C D will be 206.

Fig. 73.



77. *To inscribe four equal circles in a given circle.*—Let E O F, G O H (fig. 73) be two diameters of the given circle, intersecting each other in the centre O, at right angles to one another. Draw the tangent (**48**) A F B parallel to G H. Bisect the angles F O G, F O H by the lines O A, O B cutting the tangent in A and B. Let O A cut the circumference at P; on F O take F K equal to P A, then the circle drawn from K as a centre, with K F as radius, will touch the given circle at F, and also the lines O A, O B. Then from O as a centre, with O K as radius, describe a circle cutting the diameters in L, M, N; the circles drawn from these points as centres, and with F K or A P as radius, will be inscribed in the given circle.

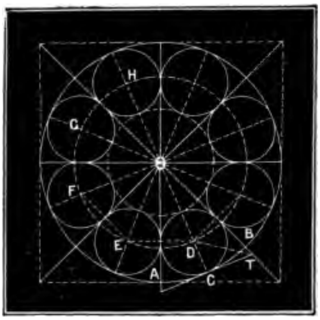
The point K can also be found by bisecting the angle O A B by the line A K.

If the radius of the given circle is 1,000 on any scale, then the length of A P or K F, the radius of each of the inscribed circles, will be 414, and that of O K 586, on the same scale.

In order to describe the small circles touching the given circle and the inscribed ones, draw K T R parallel to G H, and cutting A O in R. A circle drawn from R as a centre, with R P as radius, will touch two of the inscribed circles, and also the given one at P. Also a circle drawn from O as a centre, with radius O S equal to R P, will touch all the inscribed circles.

78. *To inscribe eight equal circles in a given circle.*—Let O (fig. 74) be the centre, O C the radius of the given circle. Divide it, as in the last problem, (**77**) into eight equal sectors, and let A O B be one of these sectors, the angle A O B being bisected by O C. Draw the tangent C T (**48**) perpendicular to O C and meeting O B in T. Bisect the angle O T C by the line D T, cutting O C in D. Then a circle described from D as a centre, with D C as radius, will touch the given circle at C, and also the lines A O, B O; this will be one of the required circles. From O as a centre, and with O D as radius,

Fig. 74.

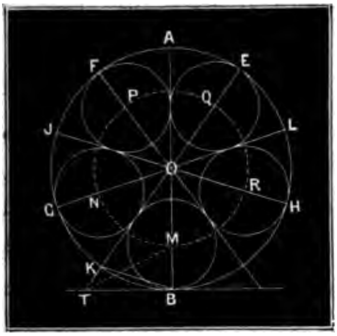


describe a circle cutting all the other diameters in E, F, G, &c., which will be the centres of the other inscribed circles, their radius being equal to D C.

If O C is represented on scale by 1,000, then the length of C D will be 277, and that of O D 723.

79. *In a given circle to inscribe five equal circles.*—Let O (fig. 75) be the centre, B O A the diameter of the given circle. Construct the isosceles triangle B O K (**25**), having each angle O K B, O B K at the base double the angle B O K at the vertex. Draw the tangent B T at right angles to B O, and produce O K to meet it at T. Bisect the angle O T B by the line T M, cutting O B in M. Then a circle described from M as a centre, with M B as radius, will be one of the inscribed circles required. Take K G, G J, J F, &c., each equal to K B, and draw the diameters through the points G, J, &c. From O as a centre, with O M as radius, describe a circle cutting these diameters in the points N, P, Q, R, which will be the centres of the other inscribed circles.

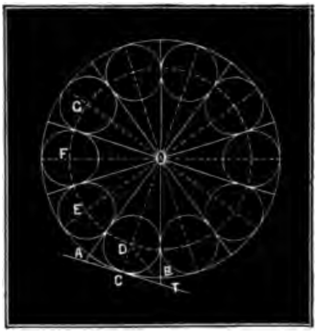
Fig. 75.



If the length O A of the radius of the given circle is represented on any scale by 1,000, then that of each inscribed circle is 370, and of O M 630.

80. *In a given circle to inscribe ten equal circles.*—Let O (fig. 76) be the centre and O A the radius of the given circle. Divide the circle into ten equal sectors, as in the last problem (**79**), and let A O B be one of those sectors. Bisect the angle A O B by the radius O C, and draw the tangent C T perpendicular thereto. Bisect the angle O T C by the line T D, cutting O C in D; then the circle drawn from D as a centre, with D C for radius, will be one of the inscribed circles required. To find the others, bisect each of the other sectors, and from O as a centre, with O D as radius, describe a circle cutting the bisecting radii in E, F, G, &c.; these will be the centres of the remaining inscribed circles, their radii being equal to D C.

Fig. 76.

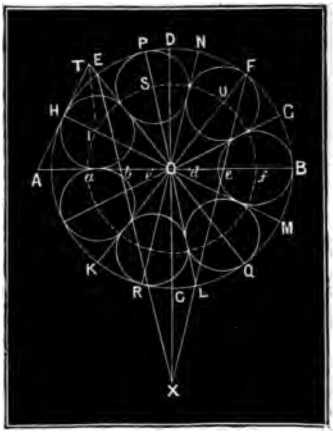


If the radius of the given circle is 1,000 on any scale, then that of each inscribed circle will be 236, and the length of O D is 794.

81. *In a given circle to inscribe (approximately) seven equal circles.*—The method of solving this problem is only an approximation, but sufficiently true for most practical purposes.

Let O (fig. 77) be the centre, A O B a diameter of the given circle. Draw D O C at right angles to A O B and produce it to X. Divide the radius into four equal parts and measure C X equal to three of those parts. Divide A B into seven equal parts at *a*, *b*, *c*, *d*, *e*, *f*. Through *b* draw X *b* E, meeting the circle at E. Draw the radius O E and bisect the angle E O A by the line H O; draw T H the tangent at H perpendicular to H O, and produce O E to meet it at T. Bisect the angle A T O by the line

Fig. 77.

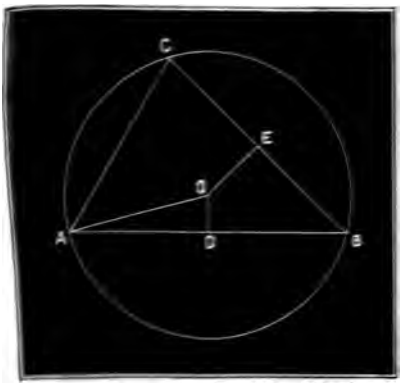


T I, cutting O H in I; then a circle described from I as a centre, with I H as radius, will be one of the inscribed circles required. To find the rest, take H P, P F, F B, &c., each equal to A E, and draw the diameters through H, P, F, &c. From O as a centre, and with O I as radius, describe a circle cutting the diameters in the points S, U, &c., which will be the centres of the other inscribed circles.

If the radius of the given circle is 1,000 on any scale, then that of each inscribed circle will be 303, and the length of the radius O I will be 697, on the same scale.

82. *To circumscribe a circle about a given triangle.*—Let A B C (fig. 78) be the given triangle. Bisect A B in D and B C in E. Draw E O and D O at right angles to B C and A B, and let them meet at the point O. Join O A. Then a circle drawn from O as a centre, with O A as radius, will pass through the vertices A, B, C of the triangle.

Fig. 78.



83. *To circumscribe a circle about any given regular polygon.*—Let F H G I K (fig. 68) be the given polygon. Bisect the two adjacent angles K F H, F H G by the lines F O, H O, meeting at O. Then the circle described from O as a centre, with O F as radius, will pass through all the vertices of the polygon.

Inscribed and Circumscribed Polygons.

84. *In a given circle to inscribe a triangle similar to a given triangle.*—Let A B C (fig. 79) be the given triangle, O the centre, and D G the diameter (fig. 80) of the given circle. Draw G E making with the diameter the angle D G E equal to the angle A C B. Draw the chord E D, and draw D F making the angle E D F equal to the angle B A C. Join E F, then the triangle D E F is similar to the given triangle, and is inscribed in the given circle.

Fig. 79.

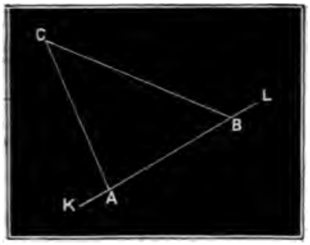
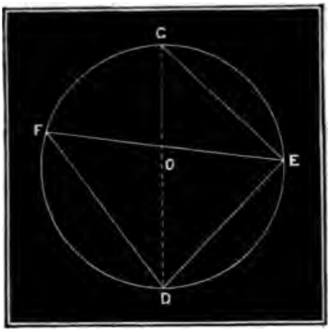
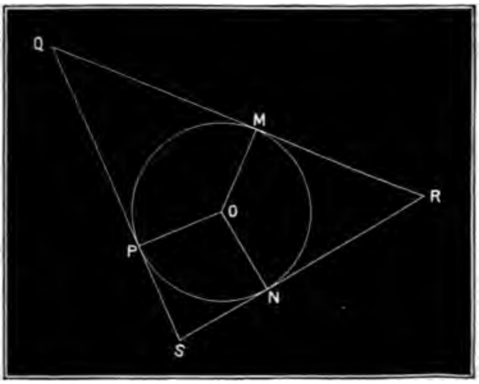


Fig. 80.



85. *About a given circle to circumscribe a triangle similar to a given triangle.*—Let A B C (fig. 79) be the given triangle, and let O be the centre (fig. 81) of the given circle, O M its radius. Produce A B to K and L; at any point M draw the tangent to the circle Q M R at right angles to the

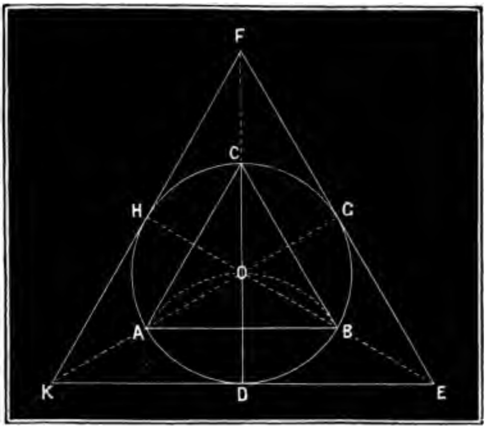
Fig. 81.



radius O M, and draw O N making with O M the angle M O N equal to the angle C B L (**7**). Draw the tangent S N R at right angles to the radius O N, and cutting Q M R in the point R. Draw O P making with O N the angle P O N equal to the angle K A C, and at P draw the tangent S P Q at right angles to O P, and cutting R M Q in the point Q. Then the triangle S R Q is similar to the triangle A B C, and is circumscribed about the given circle.

86. *In a given circle to inscribe an equilateral triangle.*—Let O be the centre (fig. 82) of the given circle, D O C a diameter. From D as a centre, with D O as radius, describe an arc cutting the circle in A and B. Join A, B, C; then A B C is an equilateral triangle inscribed in the circle.

Fig. 82.



87. *About a given circle to circumscribe an equilateral triangle.*—Let O be the centre (fig. 82), D O C a diameter of the given circle. Produce D C to F, making C F equal to the radius of the circle. From the point F draw F G E, F H K tangents (**51**) to the circle at G and H. Draw K D E the tangent (**48**) at D, and cutting the other tangents in the points E and K. Then K E F is an equilateral triangle circumscribed about the given circle.

88. *In a given circle to inscribe a square.*—Let O be the centre (fig. 67) of the given circle, E O F a diameter. Draw the diameter G O H at right angles to E O F, and draw the chords E H, H F, F G, G E. Then G E H F is a square inscribed in the given circle.

89. *About a given circle to circumscribe a square.*—Let O be the centre (fig. 67) of the given circle, E O F a diameter. Draw the diameter G O H at right angles to E O F; and draw A E B, D F C, tangents at E and F, parallel to G O H. Also draw A G D, C H B, tangents at G and H, parallel to E O F. Then A B C D is a square circumscribed about the given circle.

90. *In a given circle to inscribe a regular octagon or figure of eight equal sides.*—Let O be the centre (fig. 67) of the given circle, H O G, E O F two diameters at right angles to each other. Bisect the angles E O G, E O H by the diameters I O L, K O M, cutting the circle in I, K, L, M. Draw the chords I E, E K, K H, &c., and the inscribed octagon is completed.

91. *About a given circle to circumscribe a regular octagon.*—Let O be the centre (fig. 67) of the given circle, A B C D the circumscribed square (**89**). From A, B, C, and D as centres, with A O as radius, describe arcs cutting the sides of the square at P, Q, R, &c. Then the lines joining these points will form a circumscribed octagon.

Another method is to draw the diagonals A C, B D of the circumscribed square cutting the circle in I, K, L, M. Draw tangents P Q, R S, &c., at I, K, &c., parallel to the diagonals, and the octagon is completed.

92. *In a given circle to inscribe a regular pentagon or* *decagon.*—Let O be the centre (fig. 68) of the given circle, O F a radius. Construct the isosceles triangle O F L (**25**), having each angle at the base O F L, O L F, double the angle F O L at the vertex. Then F L is one side of an inscribed decagon or polygon of ten sides. Take L K equal to L F and draw the chord F K, which is one side of an inscribed pentagon or figure of five sides. By drawing arcs from F and K as centres, with F K as radius, cutting the circle at H and I, we obtain two more points on the pentagon; and drawing the diameter L O G, we find the fifth point G. Draw the chords F H, H G, G I, I K, and the pentagon is completed.

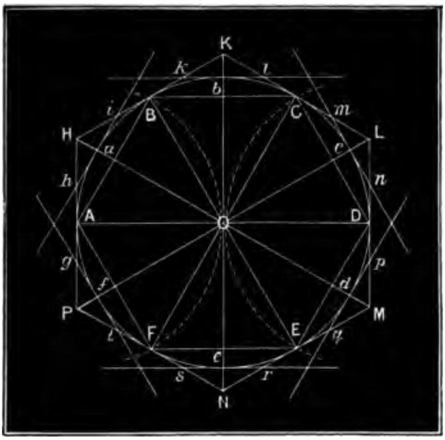
Bisect the angles F O H, H O G, G O I, I O K, by the lines O M, O N, &c., and the points L, F, M, H, N, &c., are the vertices of the inscribed decagon, and by drawing the chords F M, M H, &c., the decagon is completed.

93. *About a given circle to circumscribe a regular pentagon or decagon.*—Let O be the centre (fig. 68) of the given circle, F O P one of the diameters. Construct the isosceles triangle F O L, having each angle at its base F L double the angle at the vertex O (**25**). Draw the tangent A F E at right angles to O F, and produce O L to meet it at E; take F A equal to F E, and E A will be one side of the required pentagon. Draw the diameter L O G, and bisect the angles F O G, L O P by the diameter H O Q. Join A O and produce A O to meet the circle at I. At H, G, and I draw the tangents A H B, B G C, C I D, each equal in length to A E; and join D E. Then A B C D E is the circumscribing pentagon or figure of five sides.

To construct the circumscribing decagon, or figure of ten sides, draw tangents at L, M, N, P, and Q on the circumference, cutting the sides of the circumscribed pentagon, and the figure thus formed will be the circumscribing decagon.

94. *In a given circle to inscribe a regular hexagon, or figure of six equal sides.*—Let O be the centre (fig. 83), A O D the diameter of the given circle. From A and D as centres, and with A O as radius, describe arcs cutting the circle in B, C, E, and F. Draw the chords A B, B C, &c., and the inscribed hexagon A B C D E F is completed.

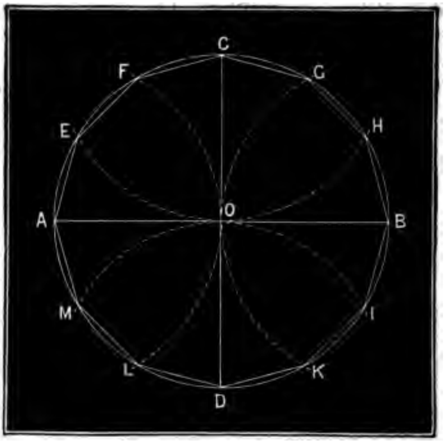
Fig. 83.



95. *About a given circle to circumscribe a regular hexagon.*—Let O be the centre (fig. 83) of the given circle, A O D a diameter. From A and B as centres, and with A O as radius, describe arcs cutting the circle in B, C, E, and F. Draw the tangents to the circle (**48**) at the points A, B, C, D, E, and F, and let them meet at the points H, K, L, M, N, and P; then the figure H K L M N P will be the circumscribed hexagon.

96. *To inscribe in a given circle a regular dodecagon, or figure of twelve equal sides.*—Let O be the centre (fig. 84) of the given circle, A O B, C O D two diameters at right angles to each other. From A, B, C, and D as centres, and with O A as radius, describe arcs cutting the circle in

Fig. 84.



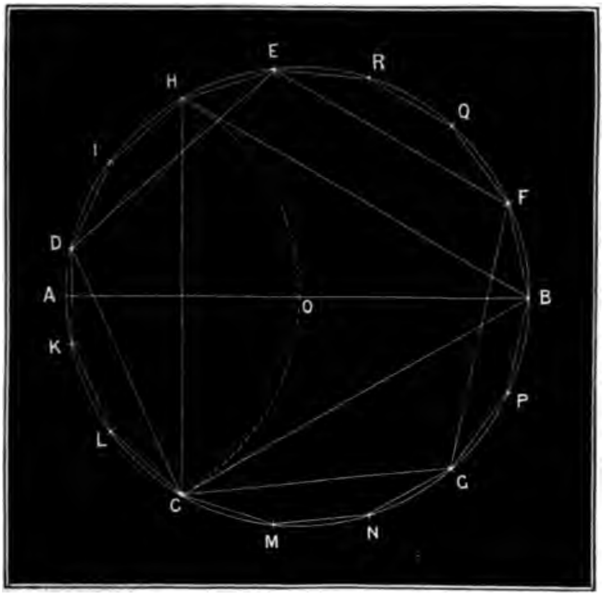
E, F, G, H, &c. Draw the chords A E, E F, F C, &c., and the dodecagon is inscribed in the given circle.

97. *About a given circle to circumscribe a regular dodecagon*—Let O be the centre (fig. 83) of the given circle. Construct the circumscribing hexagon (**95**) H K L M N P, and draw the diagonals H M, K N, L P, cutting the circumference in *a*, *b*, *c*, *d*, *e*, *f*. Draw tangents at the points *a*, *b*, &c., intersecting the sides of the circumscribing hexagon in the points *g*, *h*, *i*, *k*, &c., and a dodecagon *g h i k l*, &c., will be circumscribed about the circle.

98. *In a given circle to inscribe a quindecagon, or figure of fifteen equal sides.*—Let O be the centre (fig. 85) of the given circle, A O B a diameter. From A as a centre, and with A O as radius, describe an arc cutting the circumference in C and H. Draw the chords C H, H B, B C; then B C H is an inscribed equilateral triangle.

From C inscribe a regular pentagon (**92**) C D E F G in the circle. Draw the chords H E, F B, which are two of the sides of the quindecagon. Take F Q, E R, H I, D K,

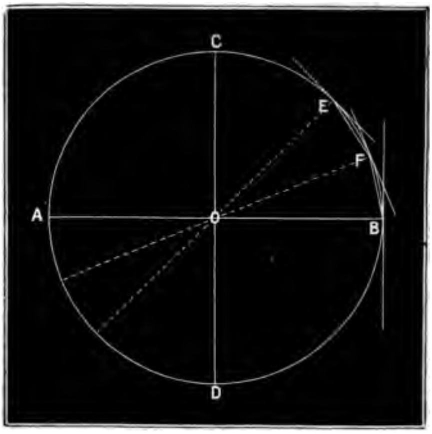
Fig. 85.



C L, C M, G N, G P, each equal to H E or F B, and draw the chords; then the regular quindecagon is inscribed in the circle.

99. *In a given circle to inscribe a polygon of any number of equal sides.*—Let O be the centre (fig. 86) of the given circle, A B and C D two diameters drawn at right angles to each other. Divide the right angle B O C into as many equal angles (**9**) as the required polygon is to have sides, and let F O B be taken equal to four of such angles. Then the chord B F will represent one of the sides of the inscribed polygon, and the remaining sides can be drawn as in the previous examples. Thus, for example, let the required polygon have sixteen equal sides. Bisect the angle B O C by the radius E O, and bisect the angle E O B by the radius F O. Then the angle F O B is equal to four-sixteenths of the right angle B O C, or to one-sixteenth of the whole circumference; and the chord B F will represent a side of the required polygon.

Fig. 86.



The number of sides which an inscribed polygon can have is limited to the number of equal parts into which a right angle can be divided (**9**). These are any power of 2 and any number being a product of 3 into a power of 2.

100. *About a given circle to circumscribe a polygon of any number of equal sides.*—First inscribe a polygon (**99**) having the same number of sides as the required polygon. Let B F, F E, &c. (fig. 86), be sides of the inscribed polygon; draw the radii O B, O F, O E, &c., and at the points B, F, E, &c., draw tangents perpendicular to the radii. Then these tangents will form a polygon of the required number of sides circumscribing the given circle.

101. *To measure the curvature of curved lines.*—Let A O B (fig. 40) be the diameter of a circle, A T the tangent at A, draw TP perpendicular to AT, and meeting the circle at the point P. The *curvature* of the circle is measured by the length PT as compared with the length AT, and is the same throughout every part of the same circle, AT being a fixed length. Also in two given circles of different radii, if AT is taken the same length in each, then PT will be proportional to the length of the radii of the circles. On account of this property that by varying the diameter of the circle it can be made to have any curvature we please, the circle is used to measure the curvature at any proposed points of other curves.

The circle which at any point of a given curve has the same tangent and curvature as the curve is called the *circle of curvature* at that point, the curvatures being in the same direction; and its radius is called the *radius of curvature* of the given curve at that point.

If the given curve coincides for a short length of arc *very nearly* with the arc AP (fig. 40) of the circle, then AO is the radius of curvature to the curve at A, and its length is found by dividing the square of the arc AP by twice the deviation PT. The length of the radius of curvature can be found by aid of the differential calculus for every point upon any given curve; and by this means we are able to ascertain whether the curvature is increasing or decreasing, and whether the curve is concave or convex. When a continuous mathematical curve changes from concavity to convexity, the radius of curvature becomes infinite at the point where the change takes place, which is called the *point of contrary flexure*, the curve becoming nearly straight about that point. As it approaches the point of flexure the curvature gets less and less, and the radius of curvature greater and greater.

CHAPTER III.

CONIC SECTIONS.

102. DEFINITIONS.—The term CONIC SECTIONS is applied by mathematicians to three curves which can be obtained from a cone having a circle for its base, by cutting it with a plane surface; the nature of the curve being determined by the direction in which the plane cuts the cone. Thus, if the plane cuts right through obliquely from side to side of the cone, the outline of the section so produced is the curve called an ELLIPSE, varying in shape according to the angle which the cutting-plane makes with the axis of the cone. Hence it is evident that any perspective representation of a circle must be an ellipse; for if we suppose the position of the eye to be the vertex, all the rays of light from the perimeter of the circle will form a conical surface, which being cut across by the plane of the picture, projects thereon an ellipse.

When the cutting-plane above mentioned is made to cut through one side only of the conical surface in a direction parallel to the other side, the outline or curve which we obtain is called a PARABOLA. Of this curve there are no varieties, there being but one parabola, larger or smaller according to the scale to which it is drawn; just as there is, mathematically speaking, but one circle.

By making the cutting-plane pass through one side of the conical surface in a direction parallel to its axis, the curve called the HYPERBOLA forms the outline of the section.

We can obtain an infinite variety of hyperbolas by varying the vertical angle or amount of slope which the cone possesses.

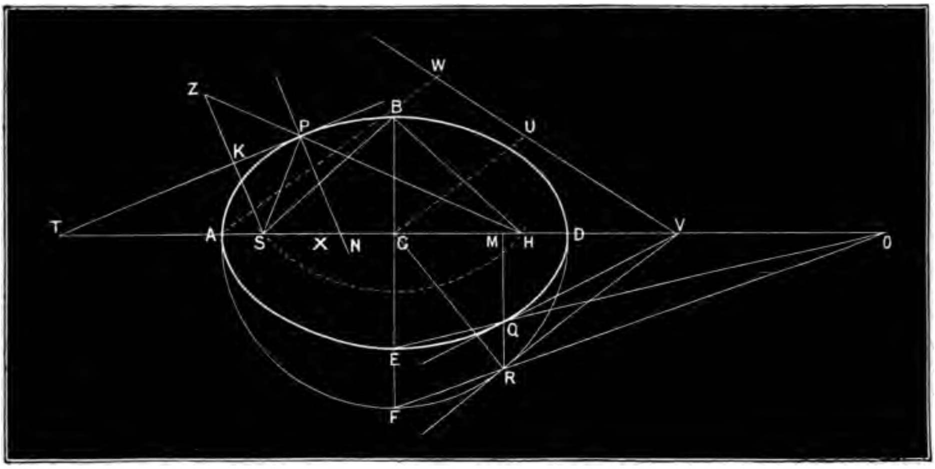
In considering, however, these three curves, their connection with the cone may be entirely lost sight of, and they can be treated as curves whose proportions and properties have been discovered purely by mathematical reasoning. Upon no other subject connected with geometry has so much been written as upon the conic sections, being curves of great importance in almost all scientific investigations. We shall, however, in the following pages confine ourselves chiefly to the consideration of such properties of these curves as render them peculiarly suitable for application to architectural design.

THE ELLIPSE.

103. DEFINITIONS.—An *Ellipse* is an oval-shaped curve entirely enclosing a surface as A B D E (fig. 87), and may be regarded as a flattened circle. Its greatest length A B is called the *major-axis,* and the greatest width B E the *minor-axis*; these axes intersect at right angles in a point C called the *centre.* If from B as a centre, with A C as radius, a circle is drawn cutting the major-axis at S and H, each of those points is called a *focus* of the ellipse; and it is a property of the curve that if lines S P, H P are drawn from the *foci* to any point P on the curve, the *sum* of the lengths of those lines is always equal to the length of the major-axis A D; the line S B is called the *radius-vector,* which may be supposed to be a moving rod having one end fixed at S and the other end always on the curve. When C B is nearly equal to C A, then S and H are near the centre;

but if C B is much less than C A, S and H are near the extremities of the major-axis, and at a distance from the

Fig. 87.



centre; consequently the ratio of C S to C A is called the *eccentricity* of the ellipse, which increases in quantity with its flatness, and decreases as it approaches nearer and nearer to a circle in form. A *tangent* is a straight line as P T, which touches the curve without cutting it; a *normal* is a right-line as P N, perpendicular to the tangent at the point of its contact with the curve.

104. *To describe an ellipse of given length of axes by continuous motion.*—Let A C D, B C E, be the given axes (fig. 87). From B as a centre, with A C as radius, draw an arc cutting A D in S and H the foci of the proposed ellipse. Fix pins at S and H, and attach thereto a string whose length, when tightened, is equal to A D. Tighten the string by means of a pencil round which it can move freely, and as the pencil moves from A towards B or E it will trace out the ellipse. The arc from A to B, or A to

E, is one-fourth of the ellipse, and is exactly equal and similar to the part from D to B, or from D to E; the axes dividing the ellipse into four equal and similar parts.

105. *To find any number of points to the circumference of an ellipse of given axes.*—Let S and H (fig. 87) be the foci of the ellipse, determined as described above (**103**). Take any point X between C and S, and from S as a centre, with A X as radius, describe a circle, and from H as a centre, with D X as radius, draw an arc cutting the circle in the point P, which will be a point upon the circumference of the required ellipse. Then by taking any number of points on the axis between C and S, we can find a corresponding number of points on the curve, in the same way as P is found. The curve can then be approximately drawn by hand through the points thus found, or by means of a straight-edge bent through the points.

106. *To draw an ellipse by help of the circumscribed circle, the length of the axes being given.*—Let A C D, B C E, be the axes (fig. 87) of the ellipse. From C as a centre, with C A as radius, describe a circle A F D. Take any point M on the major-axis and erect the *ordinate* M R, perpendicular to the axis, cutting the circle at R. Draw the chord of the circle F R and produce it to meet the axis produced in O; draw O E cutting M R at Q, then Q is a point on the required ellipse. M Q is also a *fourth-proportional* (**16**) to C F, C E, and M R. In this manner we can find any number of points on the ellipse by drawing ordinates from the axis to the circle and finding a fourth-proportional to C F, C E, and the ordinate to the circle.

The points on the ellipse can also be practically found by the use of two different scales. Let C D or C F represent 100 on any scale, and let C E measure 100 on a smaller scale; then whatever M R, the ordinate of the circle, measures on the first scale, the same will the ordinate M Q of the ellipse measure on the second scale. Any number of points can be thus determined with tolerable accuracy, and the ellipse may be drawn through them by hand. The points on the ellipse can also be found from those on the circle by help of a pair of proportional compasses set in the ratio of A C to B C.

107. *To draw a tangent at any given point on an ellipse, by help of the foci.*—Let S and H (fig. 87) be the foci of the ellipse, P the given point on the circumference. Draw S P, H P, and produce H P to Z, making P Z equal to P S; and join S Z. Bisect S Z at K, and draw P K T, which is the tangent to the ellipse at the point P.

The tangents at A and D are perpendicular to the major-axis A D and parallel to the minor-axis B C E; those at B and E are parallel to the major-axis A C D, and at right angles to the minor-axis B E.

108. *To draw a tangent at any point of an ellipse, by help of the circumscribed circle.*—Let A F D be the circumscribed circle (fig. 87) struck from the centre C; Q the given point on the circumference of the ellipse at which the tangent is to be drawn. Draw the ordinate M Q R at right angles to the axis A D, and cutting the circle at R. Draw R V the tangent to the circle at R (**48**), cutting the axis produced at V. Draw V Q which will be the tangent to the ellipse at Q.

109. *To draw a tangent to an ellipse from a given point on the major-axis produced.*—Let V (fig. 87) be the given point from which the tangent is to be drawn. Draw any line V W, making an acute angle with V A, and take V U equal to the half-major-axis D C. Join C U, and draw

A W parallel to C U. Take C M equal to U W and erect the ordinate M Q at right angles to C M; then the line V Q will be the tangent at the point Q. C M is the third proportional (**15**) to C V and C A, or C V : C A :: C A : C M.

110. *To draw the normal, or perpendicular to the tangent at any given point on an ellipse.*—Let P be the given point (fig. 87) on the ellipse, S and H the foci. Draw S P, H P, and produce H P to Z, making P Z equal to P S; join Z S, and draw P N parallel to S Z; then P N is the normal at P and is at right angles to the tangent P T (**107**) at that point.

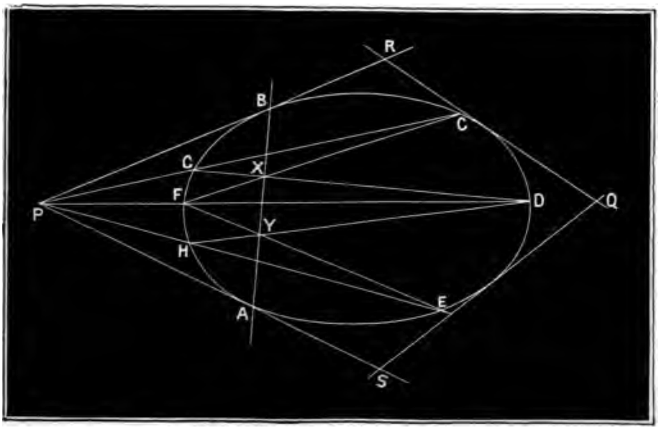
The normal can also be drawn by bisecting the angle S P H by the line P N.

At the points A, B, D, E, the axes themselves are the normals to the ellipse.

Whenever the ellipse is employed in architecture as the contour of an arch, the directions of the joints of the voussoirs must be in the normals to the curve.

111. *To draw tangents to an ellipse from any given point outside of it.*—Let P be the given point (fig. 88) outside the ellipse. From P draw three right-lines P G C, P F D,

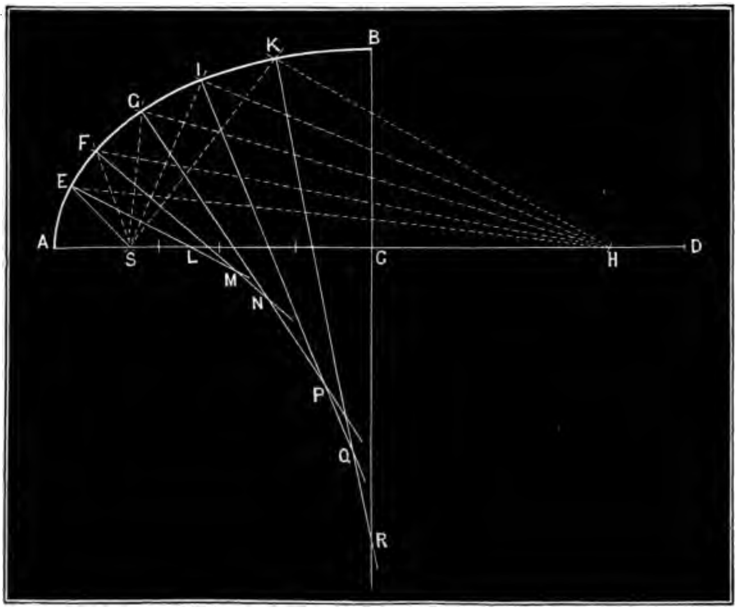
Fig. 88



P H E, cutting the ellipse in G, F, H, C, D, E. Draw the diagonals E F, D H, D G, C F, intersecting at X and Y. Draw the right-line A B passing through the points X and Y, and cutting the curve at A and B. Then the lines P A, P B will be tangents to the ellipse. In this way two tangents can be drawn from any given point outside the curve.

112. *About a given ellipse to circumscribe a trapezium of which two opposite vertices are given.*—Let P and Q (fig. 88) be the given vertices of the proposed trapezium, A B C D the given ellipse. Draw from P by the last proposition (**111**) P R, P S, tangents to the ellipse; also from Q draw Q R, Q S, tangents to the ellipse intersecting the former pair of tangents in the points R and S; then the figure P R Q S will be the trapezium required.

Fig. 89.



113. *To approximate to the outline of an ellipse by means of arcs of circles.*—Let A D (fig. 89) be the major-axis, BC the semi-minor-axis of the ellipse, S and H the foci. Find any number of points E, F, G, &c., upon the curve by the foregoing methods (**105**) and draw the normals (**110**) EL, FM, GN, &c., at those points. Let the normal at E intersect AC at L, and from L as a centre, with LA or LE as radius, describe an arc AE. Let the normal at F meet that from E at the point M, and from M as a centre, with ME or MF for radius, describe the arc EF. Let the normal at G intersect the normal at F in the point N, and from N as a centre, with NF or NG as radius, describe the arc FG. Proceeding in this way at the other points we can obtain a contour consisting of a number of arcs of circles varying in curvature which will form a near approach to the outline of an ellipse.

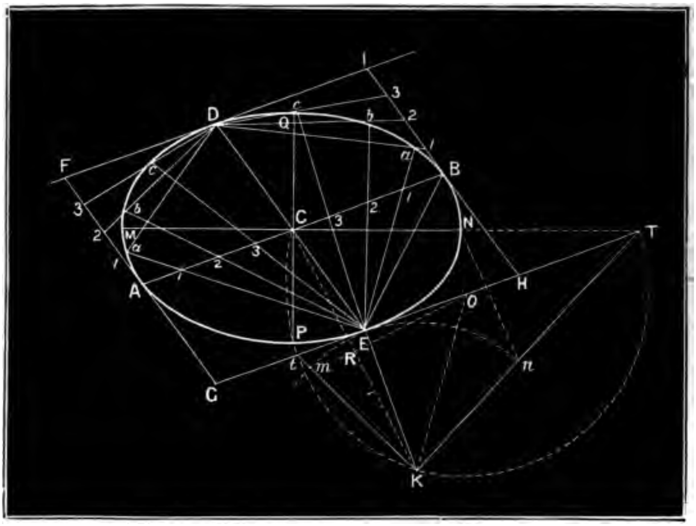
The larger the number of points that are taken on the ellipse the nearer will the resulting curve appear to approach to a true ellipse.

114. DEFINITIONS.—Any line AB (fig. 90) drawn across an ellipse and passing through the centre (C) is called a *diameter*; if a second diameter DE is drawn parallel to the tangents FAG, HBI, at the two ends of the first, the two diameters are said to be *conjugate.* Any line drawn from one diameter to the curve and parallel to the other diameter is an *ordinate.* The tangents at the two ends of any diameter are parallel to one another. Each conjugate-diameter bisects every chord of the ellipse which is drawn parallel to the other conjugate-diameter.

The tangents at the ends of a pair of conjugate-diameters form a circumscribing parallelogram FGHI to the ellipse, which is always the same in area, and is divided into four equal parallelograms by the conjugate axes. Two lines drawn from a point on an ellipse to the extremities of a diameter are called *supplemental chords*, as *a* E, *a* D.

115. *A pair of conjugate-axes being given to find points on the ellipse by means of the supplemental chords.*—Let A B, D E (fig. 90) be the given axes, C the centre of the ellipse. Divide A C, B C, into any number, as four, equal parts by the points 1, 2, 3. Also divide A F, B I, into the same number of equal parts, and join the points 1, 2, 3, on those lines with the point D. From E draw chords passing through the points on A C, B C, and cutting those drawn from D in *a*, *b*, *c*, which will be points on the perimeter of the ellipse.

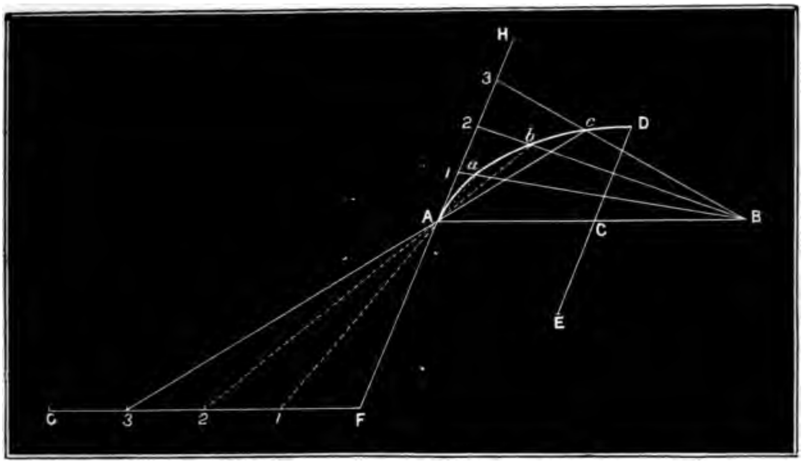
Fig. 90.



Another method.

Let A B, D E (fig. 91) be the given axes, C the centre of the ellipse. Draw H A F parallel to D E, and make A F and A H each equal to D E. Draw F G parallel and equal to A B. Divide F G into any number, say four, equal parts by the points 1, 2, 3; also divide A H into the same number of equal parts, and draw lines to B from each dividing point on A H. Through A draw lines from each of the points on F G intersecting the chords drawn from B

Fig. 91.



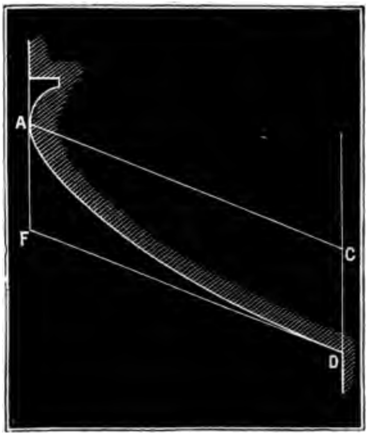
in *a*, *b*, *c*, which will be points on the circumference of the required ellipse.

116. *In a given parallelogram to inscribe an ellipse.*—Let F G H I (fig. 90) be the given parallelogram. Bisect the sides in the points A, B, D, E; and join D E, A B, by lines intersecting at C, which will be the centre of the required ellipse, A B and D E being conjugate diameters. Proceed to find points on the circumference as before described (**115**), and draw the curve touching the parallelogram at A, B, D, and E.

117. *Any pair of conjugate diameters of an ellipse being given to find the axes.*—Let A B, D E (fig. 90) be the given diameters, C the centre of the ellipse. Draw the tangent G E T parallel to A B, and erect the line E K perpendicular to E T and equal to C B. Now find the centre of the circle which shall pass through C and K and have its centre on the tangent E T; this is done by joining C K and bisecting C K at right angles by the line O R meeting the tangent at O. Join O K, then from O as a centre, with O K as radius, describe a circle cutting the tangent in the points T and *t*. Join C T, C *t*, and these will be the directions of the axes of the ellipse, since B C is a mean proportional between E T and E *t*. To find the points N and P where the axes cut the ellipse, join K T, K *t*, and on them take K *n*, K *m*, each equal to K E. Draw *n* N, *m* P, parallel to K C; then C N, C P are the lengths of the half-major and minor axes of the ellipse.

118. *To apply the ellipse to form the contour of architectural mouldings.*—Let it be required to draw an ovolo of which the tangent at one point D (fig. 92) is given, and

Fig 92.



also the greatest projection at A. Draw the vertical line A F, and also D C parallel to A F. Draw A C parallel to F D; then C is the centre of the ellipse, C A and C D semiconjugate diameters, and the ellipse can be drawn by points as described above (**115**).

Other forms of the ovolo can be drawn by taking A C, B C (figs. 93, 94), as the semi-axes, the flatness or roundness of the curves being regulated by the ratio of C A to C B.

Fig. 93.

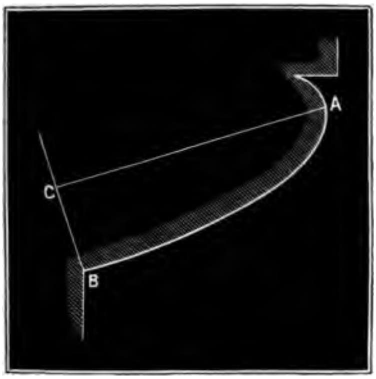
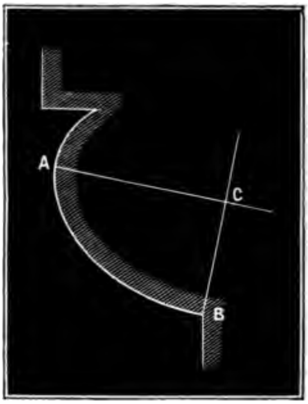
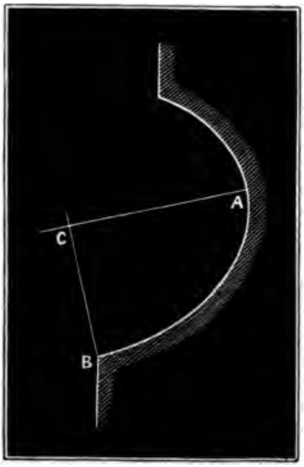


Fig. 94.



The cavetto or hollow moulding (fig. 95) can be formed out of a portion of an ellipse, C being the centre, C A and C B the semi-axes.

Fig. 95.



A portion of an ellipse D A E (figs. 96, 97) will serve to form the section of a Gothic torus or base moulding, C being the centre of the ellipse C A and C B its semi-axes.

Fig. 96

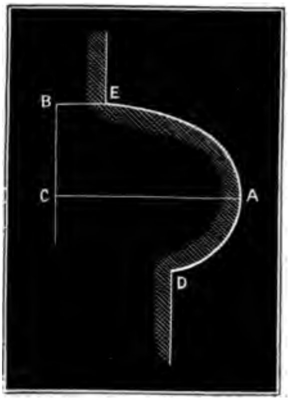
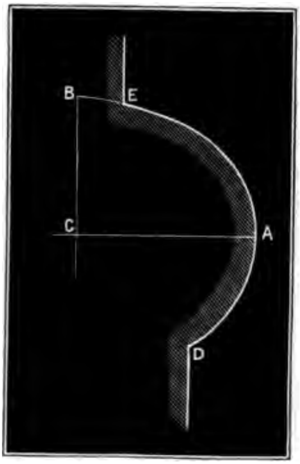
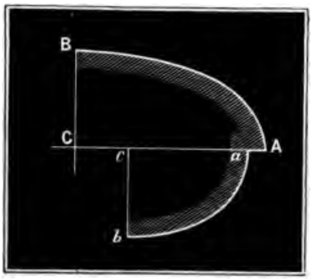


Fig. 97.



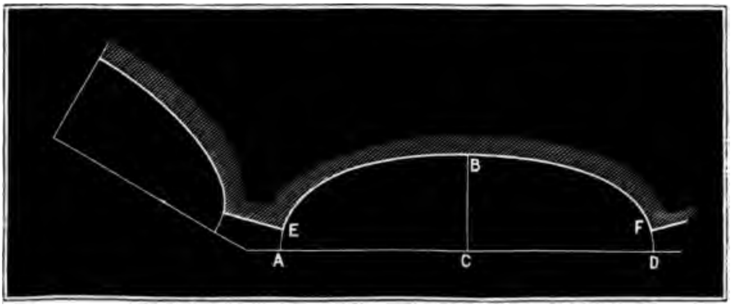
Two quadrants of different ellipses can be employed, as in fig. 98, to form a moulded Gothic string-course; C being the centre, and C A, C B the semi-axes of one ellipse; *c* the centre, and *c a*, *c b* the semi-axes of the other ellipse.

Fig. 98.



In fig. 99 is shown how the ellipse can be applied to

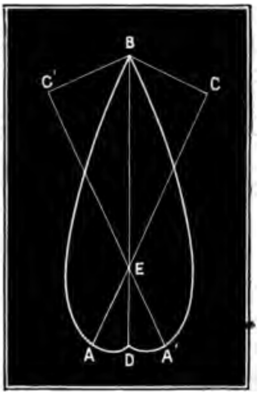
Fig. 99.



form the sections of flutings for columns; E B F being the outline of the fluting rather less than half an ellipse, whose centre is at C, and whose semi-axes are C A, C B.

119. *To apply the ellipse to form the contour of leaf ornaments.*—Let B D (fig. 100) be the axis of the leaf; draw A C, A′ C′ intersecting the axis at E, and making equal angles with it. Draw B C perpendicular to A C, B C′ to A′ C′; then A C, B C are the semi-axes of an ellipse, and A′ C′, B C′ those of an equal and similar ellipse. The ellipses B A D, B A′ D can be drawn by either of the previous methods (**104, 105**), and the contour of a leaf will be formed.

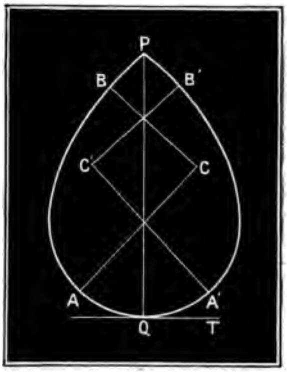
Fig. 100.



Let P Q (fig. 101) be the axis of the leaf, Q T a line at right angles to Q P. Let Q T be the tangent to both the arcs of the ellipses, of which C and C′ are the centres, C A, C′ A′ the semi-major axes, making equal angles with P Q; B C, B′ C′ the semi-minor axes. Then the contour of the leaf can be drawn as before, Q T being the tangent to both the ellipses at Q.

In this way any variety of outline can be obtained for purposes of ornamentation.

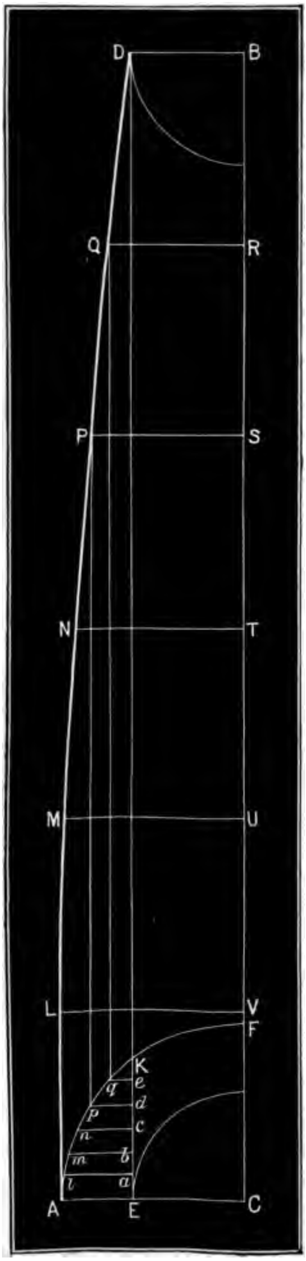
Fig. 101.



120. *To apply the ellipse to form the entasis of a column.*—Let B C (fig. 102) be the axis of the column, A C its half-diameter at base, B D the half-diameter at summit.

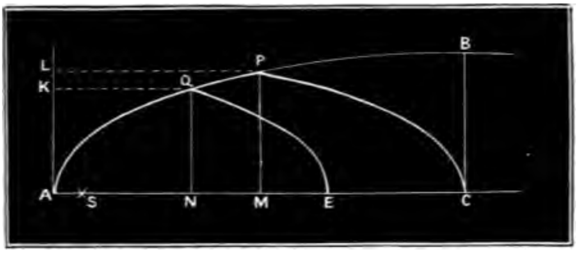
Draw D E parallel to B C, then A E is the difference between the top and bottom radii. From C as a centre, with C A as radius, describe a circle cutting D E at K. Divide B C into any number of equal parts at R, S, T, &c., and draw R Q, S P, &c., at right angles to B C. Divide E K into the same number of equal parts as B C is divided into, at the points *a*, *b*, *c*, &c., and draw *a l*, *b m*, &c., at right angles to E K, and cutting the circle in the points *l*, *m*, &c. Through *l*, *m*, *n*, &c., draw lines parallel to B C intersecting those through R, S, T, &c., at the points Q, P, &c.; then Q, P, &c. will be points upon the circumference of an ellipse, and the curve can be drawn by bending a straight-edge through them.

Fig. 102.



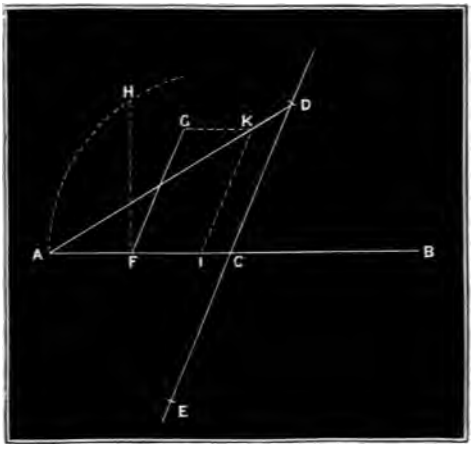
121. *To apply the ellipse to form the outline of a Tudor arch.*—Let A C, B C (fig. 103) be the semi-axes of an ellipse of considerable eccentricity, B C being greater than the vertical height of the proposed arch from the springing, and A C greater than its half-span. Draw A L at right angles to A C, and let A K or A L be the height of the arch, A N or A M its half-span. Draw N Q or M P at right angles to A C, and K Q, L P parallel to A C; then the arc A Q or A P forms one side of the arch required. The opposite side, Q E, or P C, will be drawn by repeating A Q, or A P.

Fig. 103.



By this method, a continuous curve is obtained from the springing to the vertex, the change of curvature taking place gradually throughout, and not suddenly, as is the case when the arch is struck by two different arcs of circles, forming an abrupt change in the rate of curvature at their point of junction.

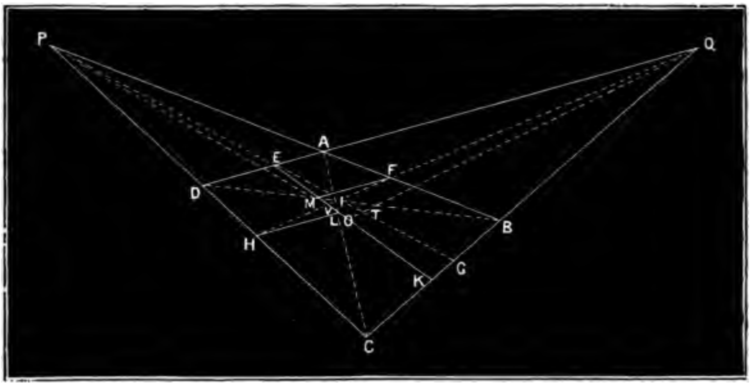
Fig. 104.



122. *A diameter of an ellipse and an ordinate being given, to find the conjugate diameter.*—Let A B (fig. 104) be the given diameter, F G the given ordinate. Bisect A B in C. At F, draw F H at right angles to A B; and from C as a centre, with C A as radius, describe a circle cutting F H in H. Take A I, equal to F H, and draw I K parallel and equal to F G. Draw D C E parallel to F G, join A K, and produce it to meet D E at D. Make C E equal to C D, and D E is the conjugate diameter required.

123. *To inscribe an ellipse in a given trapezium.*—Let A B C D (fig. 105) be the given trapezium. Produce the sides to meet at the points P and Q. Draw the diagonals intersecting at I. Through I draw P E I G, Q F I H; then the ellipse will touch the sides of the trapezium at the points E, F, G, H. Bisect E G at T, and F H at V. Draw P V O, Q T O intersecting in O; then O is the centre of the required ellipse.

Fig. 105.



Join E O, and produce E O to K, making O K equal to O E. Then E K is a diameter of the ellipse, and H L, M F, drawn parallel to A D, are ordinates. Therefore the conjugate diameter can be found by help of the last proposition (**122**), and the ellipse drawn by the method before described (**115**).

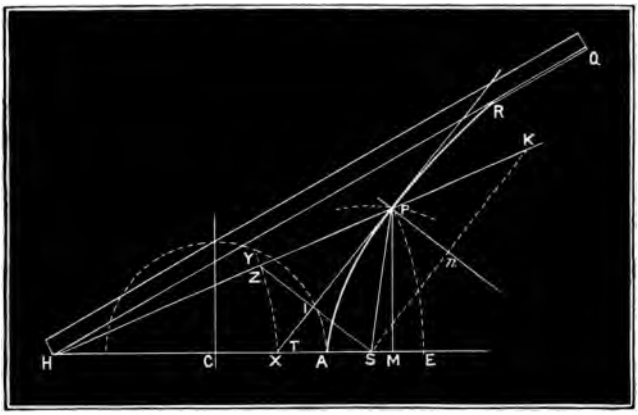
This method is of use in drawing a perspective representation of a circle, the trapezium being the representation of a square circumscribing the circle. The point K is not necessarily upon the line B C.

THE HYPERBOLA.

124. DEFINITIONS.—The name *hyperbola* is given to a class of curves which are not enclosed, or that do not enclose a space. These curves have two branches, as A L, A *l* (fig. 107), each of which start from the same point, A, and go off to infinity; the two branches are exactly alike, and are divided from each other by a right-line, C S, passing through the *vertex*, A, of the curve, which is the point where the two branches meet; this line, C S, is called the *axis* of the curve, and lines S L, S *l*, drawn at right angles to the axis, cut the two branches of the curve at equal distances therefrom. The hyperbola has two *foci*, S, H (fig. 106), and the point C, halfway between the foci, is called the *centre*. The ratio of the distance of the centre from the focus S, to its distance from the vertex A or C S : C A, is called the *eccentricity* of the curve, which is increased as A is moved towards C, and decreased as A is moved away from C; thus (fig. 107) there are three hyperbolas of different degrees of eccentricity, but having a common centre and focus. Any line drawn from the focus S to the curve, as S P (fig. 106), is called the *radius-vector.* Two lines intersecting at the centre C (fig. 107), and making equal angles with the axis C S, as C B, C *b*, to which the curve is continually approaching without ever reaching, are called *asymptotes.* When the asymptotes make a right-angle with each other, the curve to which they belong is called the *rectangular-hyperbola.* The ordinate drawn through the focus S, as S L (fig. 107), at right angles to the axis, is called the *latus-rectum.* Any distance measured along the axis, either from the centre C or the vertex A, is called an *abscissa.*

125. *To draw an hyperbola by continuous motion.*—Draw a straight line, H C S (fig. 106), as an axis, and let H S be the given distance apart of the foci, C the centre, A the given vertex. Between A and C take A X, equal to A S. Let a ruler, as H Q, be made to revolve about a pin at one end, H, in the axis; and from H measure on H Q the length H Y, equal to H X. At Q, fasten a thread of length equal to Q Y, and attach the other end to a pin at the focus S. Let the ruler lie upon the axis, and the thread be tightened by a pencil pressed against the edge of the ruler. Then, as the ruler revolves about the point H, the pencil keeping the thread tight will mark out the curve beginning at the vertex A.

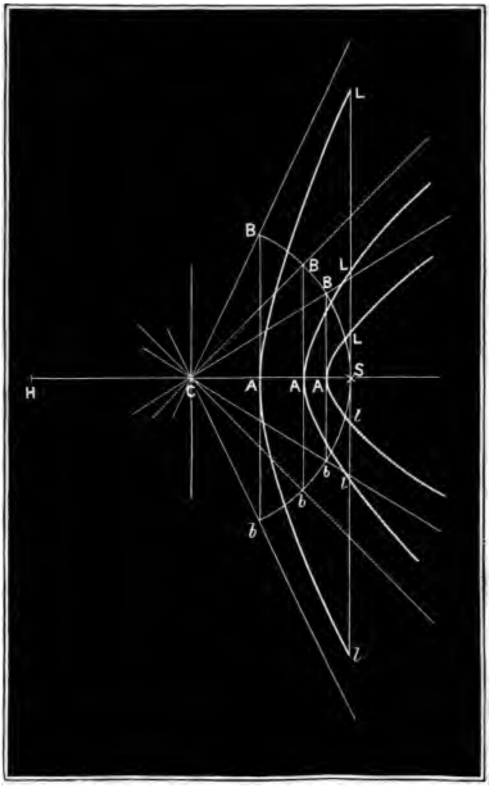
Fig. 106.



This method is derived from the property of the hyperbola that the difference between the focal distances of any point P on the curve are always the same; that is, H P – S P is always equal to H X or H Y.

126. *To find any number of points upon the contour of a hyperbola, whose vertex and foci are given.*—Let H S (fig. 106) be the axis, H and S the given foci, A the given vertex. Make A X equal to A S, take any point E on the axis to the right of the vertex A, and from S as a centre, with X E as radius, describe a circle; also from H as a centre, with H E as radius, draw an arc cutting the circle at the point P, which will then be a point upon the required curve. In the same manner we can find any number of points on the curve, by taking different points along the axis from A, and proceeding as before. Having found a sufficient number of points on the curve, we can draw the curve itself by hand, or by bending a ruler through those points.

Fig. 107.

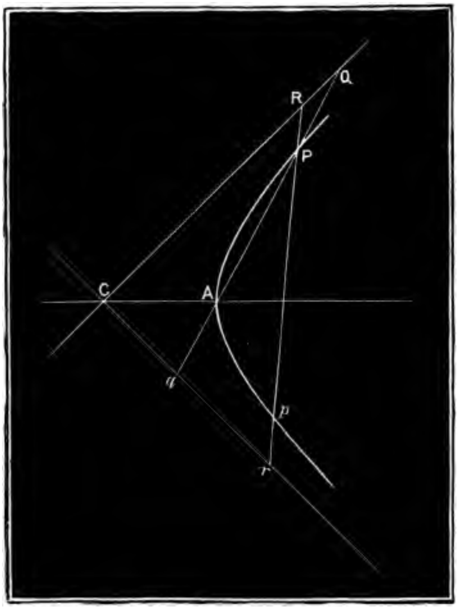


127. *To draw the asymptotes of a hyperbola, whose vertex and foci are given.* — Let H C S be the given axis (fig. 107), H and S the foci, C the centre, A the given vertex. Draw *b* A B at right angles to A C, and from C as a centre, with C S as radius, describe a circle cutting *b* A B in the points B, *b*. Draw the right-lines C B, C *b*, which will be the asymptotes of the hyperbola *l* A L.

In the rectangular-hyperbola, A B is equal to A C, and the asymptotes make a right angle with each other at the centre C.

128. *To find points on a hyperbola by means of the asymptotes.*—Let C A (fig. 108) be the axis, C R, C *r* the asymptotes found as described above (**127**). Draw any line *q* A Q making an acute angle with the axis, and cutting the asymptotes in Q and *q*. Take Q P, equal to A *q*, then P is a point on the hyperbola. So also by drawing any other lines through the vertex and intersecting the asymptotes, we can find as many points as we please upon the re-

Fig. 108.



quired curve, which can then be drawn in by hand, or by bending a ruler through those points.

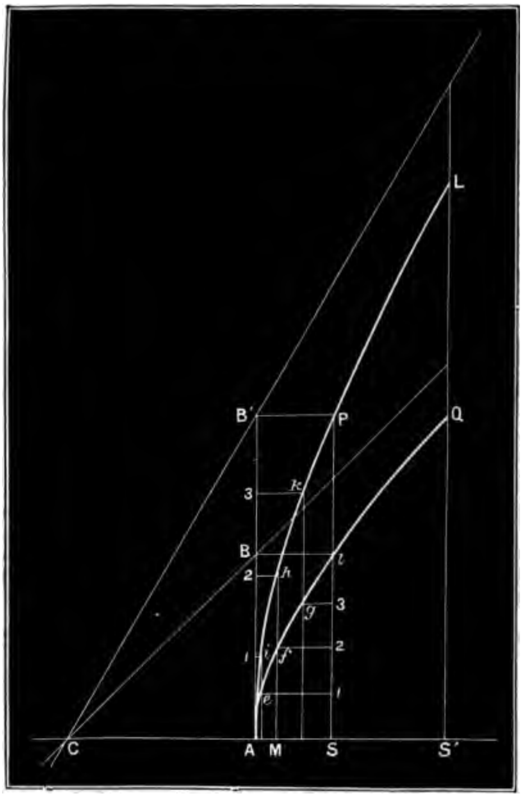
Having determined one point P on the part of the curve above the axis, we can find points on the part below by drawing any line, R P *r*, cutting the asymptotes in R and *r*. Take *r p*, equal to R P, then *p* is a point on the lower curve; and in the same way, by drawing other lines through P or A intersecting the asymptotes, any number of points on the lower curve can be found.

The two portions of the hyperbola, above and below the axis, are exactly equal and similar, so that when one is drawn the other can be drawn from it.

129. *To draw any hyperbola by means of the rectangular hyperbola.*—Let C A (fig. 109) be the axis, C the centre, and A the vertex of the required curve. Draw A B at right angles to A C, and equal thereto, then the line C B is an asymptote of the rectangular hyperbola, and taking C S equal to C B, S will be one of its foci. Let C B′ be a given asymptote of the required hyperbola, draw A B′ at right angles to A C, and make C S′ equal to C B′, then S′ is the focus of the required curve. Find points on the rectangular hyperbola, as *f*, *l*, Q, by any of the previous methods, and draw the ordinates, M *f h*, S *l* P, S′ Q L. Then, by taking M *h* : M *f*, S *l* : S P, S′ Q : S′ L in the ratio of A B′ : A B, we find corresponding points *h*, P, L, on the required curve. This can be done geometrically (**16**), or by using proportional compasses or a scale of parts.

Suppose it be required to find points on the hyperbola from A to P, draw the ordinate S *l* P, which will be equal to A B′; divide S *l* into any number of equal parts at 1, 2, 3, &c., and also divide S P or A B′ into the same number of equal parts at 1, 2, 3, &c. Draw horizontal lines through the dividing points on S l, as f 2, meeting the rectangular hyperbola at the point f; then an ordinate M f will, if produced, meet the horizontal h 2 in the point h, which will be a point on the required hyperbola.

Fig. 109.



130. *To draw any hyperbola by means of a scale of parts.*—Let A X (fig. 110) be the axis, A the vertex of the hyperbola. Measure the abscissæ by any scale from the vertex A, according to the numbers marked along the axis A X, or shown in the following table. Then by measuring the ordinates on the *same* scale, we obtain points upon the rectangular hyperbola. If we use a *different* scale for the ordinates to that used for the abscissæ, we obtain points upon any other hyperbola we may require.

Fig. 110.

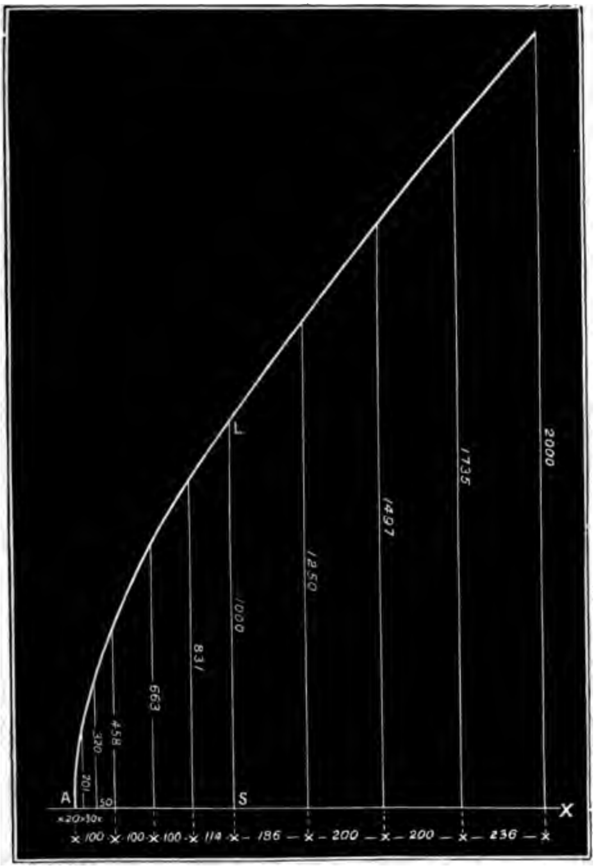
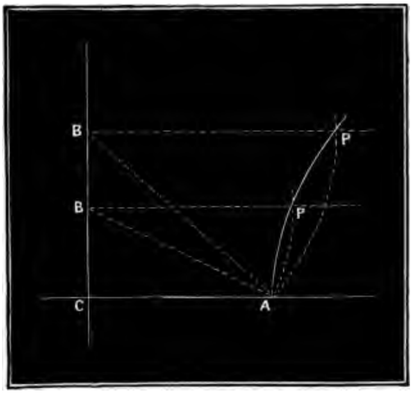


Table of Ordinates of a Rectangular Hyperbola to the Corresponding Values of Abscissæ.

Abscissæ	Ordinates	Abscissæ	Ordinates	Abscissæ	Ordinates
10	142	600	1250	2000	2829
20	201	800	1497	2162	3000
50	320	1000	1735	2400	3250
100	458	1236	2000	2600	3460
200	663	1400	2182	2800	3666
300	831	1600	2400	3000	3872
414	1000	1800	2615	3123	4000

131. *To find points on the contour of a rectangular hyperbola.*—Let C A (fig. 111) be the axis, C the centre, and A the vertex of the curve. Draw C B at right angles to C A; take any point B thereon, and join B A. Draw B P parallel to C A, and from B as a centre, with B A as radius, describe a circle cutting B P in P; then P is a point on the rectangular-hyperbola.

Fig. 111.

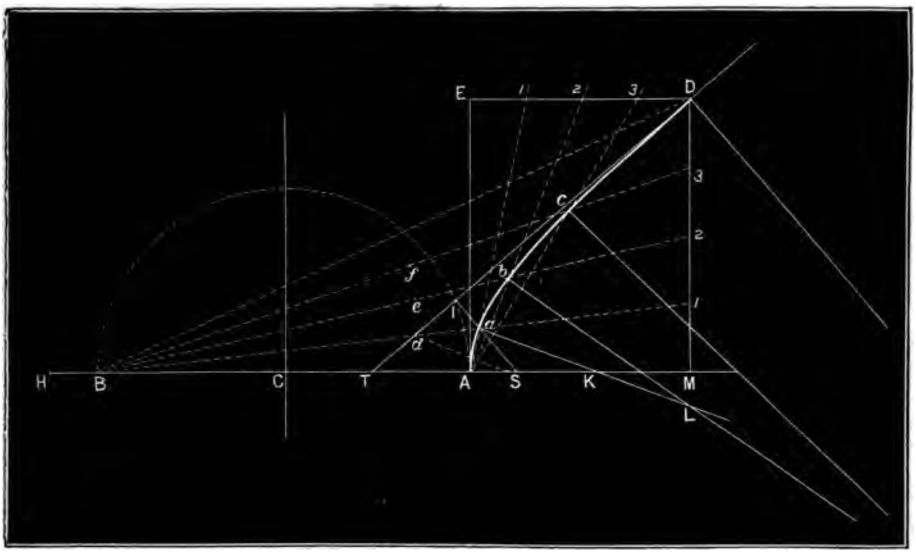


By taking any number of points on the vertical C B, we can in the same manner find as many points as we please upon the required curve.

132. *To find points on the contour of any hyperbola by the ruler only.*—Let C A (fig. 112) be the axis, C the centre, and A the vertex of the required curve. Take C B equal to C A. Suppose the curve is required to pass through a given point D; draw D M at right angles to the axis, and draw A E parallel to M D, E D parallel to A M. Divide

E D and M D into the same number of equal parts by the points 1, 2, &c. Draw the lines B 1, B 2, &c., intersecting the lines A 1, A 2, &c., in the points *a*, *b*, &c., which will be upon the contour of the required curve.

Fig. 112.



133. *To draw the tangent at any given point of a hyperbola.*—Let H C S (fig. 106) be the axis, H and S the foci, P the given point on the curve. Draw S P, H P, and produce H P to K, making P K equal to P S. Join S K, then P T drawn parallel to S K is the tangent at the point P.

The tangent can also be drawn by bisecting the angle H P S by the line P T.

Another way is to find C T a third proportional (**15**) to C M and C A, P M being the ordinate at P perpendicular to the axis. Then T P will be the tangent at P.

134. *To draw a tangent to a hyperbola from any point upon the axis outside the curve.*—Let T (fig. 106) be the given point on the axis, C the centre, A the vertex. Find

C M a third proportional (**15**) to C T and C A; draw the ordinate M P at right angles to the axis, and meeting the curve at P. Then the line T P will be a tangent to the curve at the point P.

135. *To draw a normal at any given point on a hyperbola.*—Let P (fig. 106) be the given point on the hyperbola whose foci are H and S. Draw H P, S P, and take P Z equal to P S; join Z S, and draw P*n* parallel to Z S; then P*n* is the normal at P, and is perpendicular to the tangent P T.

Another way is to produce H P to K, making P K equal P S, joining K S, and bisecting K S at *n*, then P*n* is the normal at P.

It can also be drawn by bisecting the angle S P K, by the line P*n*.

Whenever a hyperbola is used as the outline of an arch, the joints of the voussoirs must be in the normals to the curve.

136. *To find the foci of a given hyperbola.*—Let H A (fig. 112) be the axis of the hyperbola, C the centre, A the vertex, D a given point on the curve, M D an ordinate perpendicular to A M. From C as a centre, with C A as radius, describe a circle. Find C T, a third proportional (**15**) to C M and C A, and draw the tangent T D, cutting the circle at the point I. Draw I S at right angles to T D, and cutting the axis at S; then S is a focus of the hyperbola. To find the other focus, take C H equal C S, and H is the further focus of the curve.

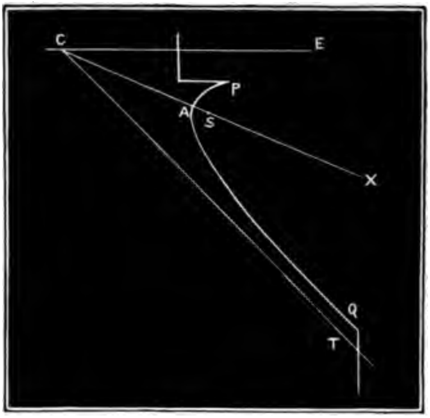
137. *To approximate to the contour of a hyperbola by means of arcs of circles.*—Let A C (fig. 112) be the axis of the hyperbola, A the vertex, S the focus, *a*, *b*, *c*, &c., points found upon the curve by the previous method (**132**).

Find the normals (**135**) at *a*, *b*, &c., by taking *a d* equal to *a* S, *b e* equal to *b* S, and drawing *a* K parallel to S *d*, *b* L parallel to *e* S, and so on. Let the normal at *a* cut the axis at K, and from K as a centre, with K A or K *a* as radius, describe the arc A *a*. Let the normal at *b* cut the normal at *a* in the point L, and from L as a centre, with L *a* or L *b* as radius, draw the arc *a b*. Proceeding in this way we can obtain a series of small arcs of circles which will form a tolerably near approach to an arc of a hyperbola; and the closer the points, *a*, *b*, &c., are taken to each other, the nearer will the resulting curve be to a hyperbola.

138. *To apply the hyperbola to form the entasis of a column.*—Let A Q (fig. 109) be a rectangular hyperbola (**131**), having C S for its axis, A for its vertex, and S its focus. Let A S be the difference between the radii at the top and bottom of the shaft of the column, S P the vertical height of the shaft. Draw the vertical A B B′, and the horizontal P B′. Let S P cut the rectangular hyperbola at the point *l*, and divide S *l* into any number of equal parts, and also A B′ into the same number of equal parts at the points, 1, 2, &c. Draw the horizontals 1 *e*, 2 *f*, &c., cutting the rectangular hyperbola at *e*, *f*, &c., and draw the ordinates through those points, producing them to meet horizontals through the divisions on A B′ in the points *i*, *h*, &c.; then these will be points on the required curve through which it can be drawn by bending a straight-edge. It is not necessary to draw the contour of the rectangular hyperbola, but only to find as many points upon it as we require to find on the entasis hyperbola.

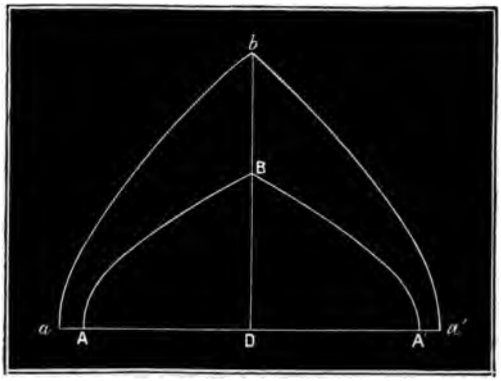
139. *Application of the hyperbola to form the outline of mouldings and other architectural forms.*—The hyperbola may be used to form an ovolo in classic architecture (fig. 113); taking C A X as the axis, C E and C T as the asymptotes, S the focus; then the curve P A Q can be drawn by any of the previous methods.

Fig. 113.



A pointed arch (fig. 114) can be formed by two equal arcs of a hyperbola meeting at the vertex B or *b*, A D being

Fig. 114.



the axis, and A or *a* the vertex of the curve; when used as a stone arch, the voussoirs must have their joints in the directions of the normals to the curve (**135**).

Leaf ornaments (figs. 115, 116) can be formed by arcs of hyperbolas. Let C D be the axis of the leaf, dividing it into two equal parts, A X the axis of a hyperbola, D A C (fig. 115) having its tangent D T at D perpendicular to C D; this will form one side of the leaf, and by reversing the outline we get the other side, D A′ C having A′ X′ for its axis, A X and A′ X′ cutting C D at equal angles. Any variety of form can be obtained; thus the curve may be turned in at D (fig. 116), and not have its tangent at that point perpendicular to D C.

Fig. 115.

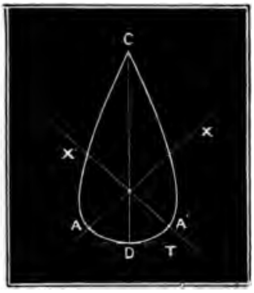
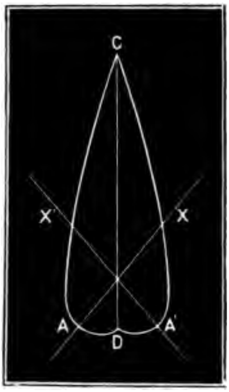


Fig. 116.



THE PARABOLA.

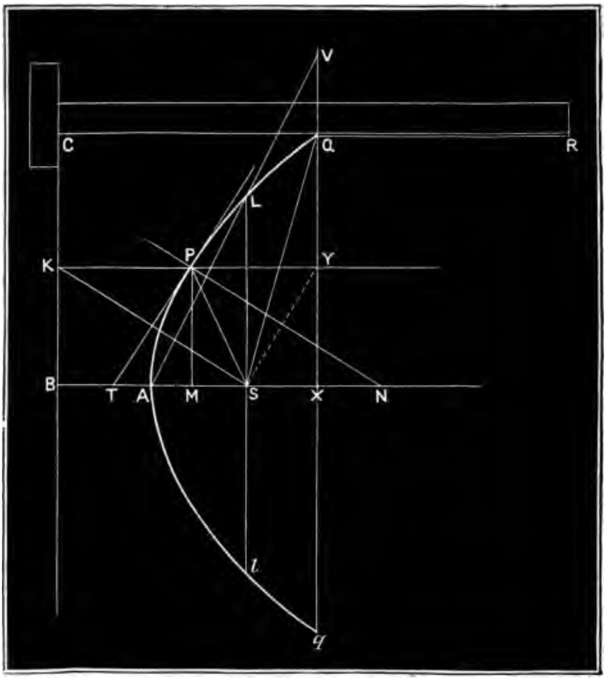
140. DEFINITIONS.—The *parabola* is a single curve, which is unenclosed, and consists of two equal branches, A Q, A *q* (fig. 117), divided by a straight line, B A X, called the *axis*, which it cuts at right-angles in a point A called the *vertex*. It has only one *focus* S situate within the curve, and the double ordinate L S *l* is called the *latus-rectum* and is equal to four times the length of S A, so that S *l* or S L equals twice S A.

If we produce the axis to B outside the curve, and take A B equal to A S, then a line B C drawn perpendicular to

B A is called the *directrix*; and if P is any point on the curve, and P K is drawn parallel to A B to meet the directrix at K, then P K is equal to P S. The line S P is called the *radius-vector*, A M is the abscissa, M P the *ordinate* of the point P on the curve. The parabola differs from the ellipse and hyperbola in having no *centre*; it may, in fact, be considered as an ellipse whose axis is of an infinite length. The *curvature* of the parabola is greatest at the vertex, and decreases as it recedes therefrom.

141. *To draw the parabola by continuous motion.*—Let B A X be the axis, A the vertex, B C the directrix, S the

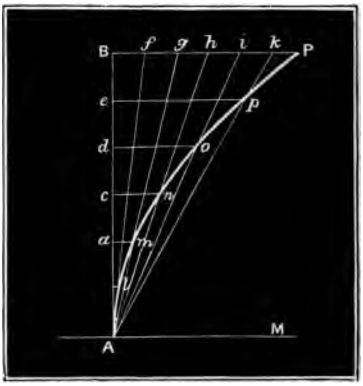
Fig. 117.



focus; A S being equal to A B. Fix a straight-edge against B C, or let B C be the edge of a drawing-board, at right angles to B X. Let a T-square C R move from the axis along B C, having a thread of length equal to R C attached at one end to R, the other end being fixed to a pin at S. If the string is kept tight by the point of a pencil pressed against the T-square as it moves upwards or downwards from the axis, the pencil will mark out the parabola.

142. *To find points on the contour of the parabola.*—Suppose the axis A M (fig. 118) and the vertex A to be given, and also a point P through which the curve is to pass. Draw A B perpendicular to A M, P B parallel to A M. Divide A B into any number of equal parts by the points *a*, *b*, &c.; and also divide B P into the same number of equal parts at *f*, *g*, &c. Draw the lines A *f*, A *g*, &c., cutting horizontals through *a*, *b*, &c., in the points *l*, *m*, &c.: these will be points on the required curve.

Fig. 118.

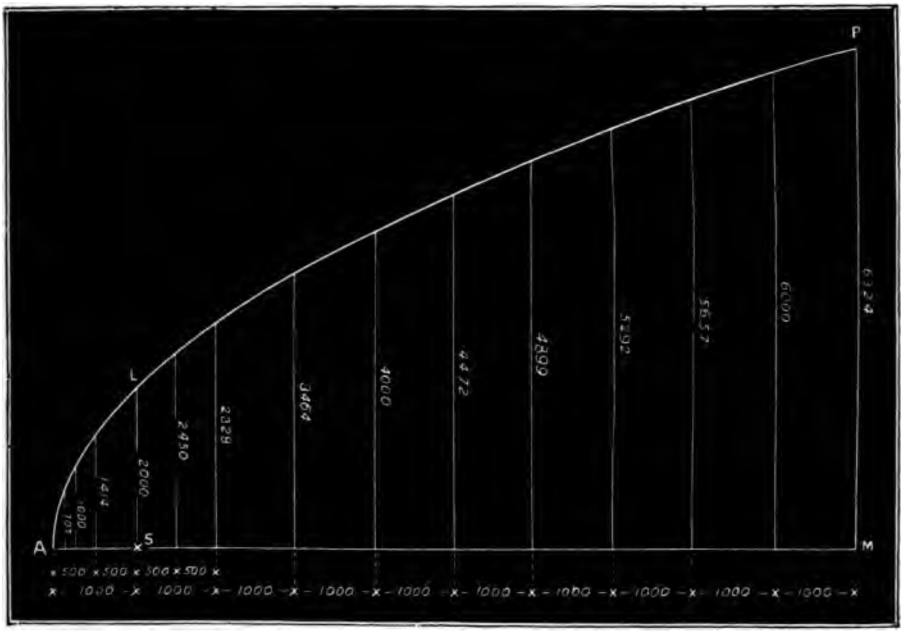


143. *To find the lengths of ordinates to a parabola by means of a scale of parts.*—Let A M (fig. 119) be the axis, A the vertex, S the focus. If we measure S A on any scale as 1000, then the ordinate S L will be 2000 on the same scale. The following table shows the relative proportions of the abscissæ and their corresponding ordinates:—

Table of Abscissæ and Ordinates of the Parabola.

Abscissæ	Ordinates	Abscissæ	Ordinates	Abscissæ	Ordinates
10	200	500	1414	5000	4472
20	283	1000	2000	6000	4899
50	448	1500	2450	7000	5292
100	632	2000	2828	8000	5657
125	707	3000	3464	9000	6000
250	1000	4000	4000	10000	6324

Fig. 119.



144. *To draw a tangent at any point of the parabola.*—Let P (fig. 117) be the given point, draw the ordinate P M perpendicular to the axis A X. Take A T on the axis produced outside the curve, equal to A M; then T P is the tangent at the point P.

Another method.

Let S be the focus, join S P, and draw P K parallel to

A X; bisect the angle S P K by the line P T, which is the tangent required. P T also bisects the line S K, P K being equal to P S.

145. *From a given point on the axis outside the parabola to draw a tangent.*—Let T be the given point (fig. 117); take A M equal to A T, and draw M P at right angles to A M, meeting the curve at P; then P T is a tangent to the curve.

146. *To draw a normal at any given point on the parabola.*—Let P (fig. 117) be the given point, A X the axis, S the focus. Join S P, and draw S Y parallel to the axis A X; bisect the angle S P Y by the line P N, which is the normal to the curve at P, and is at right angles to the tangent P T.

Another method.

Let B K C be the directrix, perpendicular to the axis, A B being equal to A S. Draw K P parallel to the axis, and join S K. Then the normal is found by drawing P N parallel to S K, P K being equal to P S.

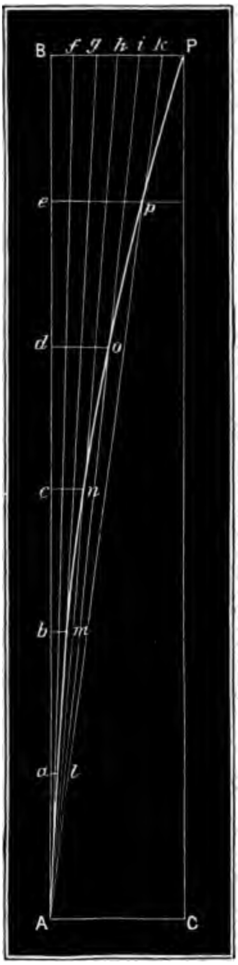
The normal can also be drawn without finding the focus, by first drawing the tangent at P (**144**), and then drawing P N perpendicular to P T.

Another method.

Let M P be the ordinate at P, at right angles to the axis; measure M N, equal to S B, or to twice S A, S being the focus; then N is the point where the normal cuts the axis, and N P is the normal required.

147. *To apply the parabola to form the entasis of a column.*—Let A B (fig. 120) be the given height of the column, A C the difference between its radii at top and bottom. Draw B P at right angles to A B, and divide A B and B P into the same number of equal parts. Draw the lines A *f*, A *g*, &c., cutting the lines *a l*, *b m*, &c., perpendicular to A B, in the points *l*, *m*, &c. These will be points on the required curve, through which it can be drawn by bending a ruler or lath. The curve thus drawn is part of a parabola (**142**), having A for its vertex.

Fig. 120.



148. *To find the focus of a given parabola.*—Let A X (fig. 117) be the axis, A the vertex, P a given point on the parabola. Draw P M perpendicular to the axis, and take A T, equal to A M. Draw the tangent T P, and also the line P Y parallel to A X.

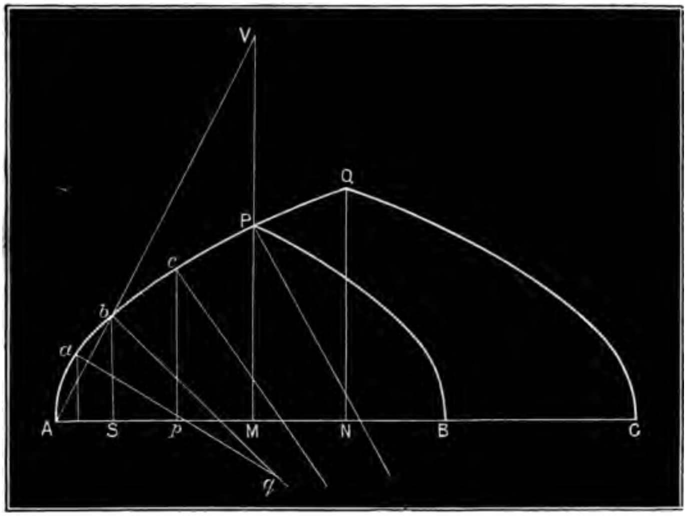
Draw the normal P N at right angles to P T. Draw P S, making with P N the angle S P N, equal to the angle N P Y; then if P S cuts the axis at S that point is the required focus.

Another method.

Let Q be a point (fig. 117) upon the curve, draw X Q perpendicular to A X, and produce it to V, making X V equal to twice X A. Join A V, and let the line A V cut the curve at the point L. Drop the perpendicular L S upon A S, and S will be the focus of the parabola.

149. *To apply the parabola to form the outline of a Tudor arch.*—Let A B C (fig. 121) be the springing line of the arch, A M or A N the half span, M P or N Q its height at the vertex. Through P or Q draw the parabola A P Q, having A for its vertex (**142**); then A P or A Q will form one-half of a flat-pointed or Tudor arch; and by repeating the curve on the opposite side of the centre line M P or N Q we obtain the other half P B or Q C of the arch. The joints of the voussoirs must all be in the directions of the normals (**146**) to the curve. Take M V, equal to twice M A, join A V, cutting the curve at *b*, draw *b* S perpendicular to A M, then S is the focus of the parabola (**148**).

Fig. 121.

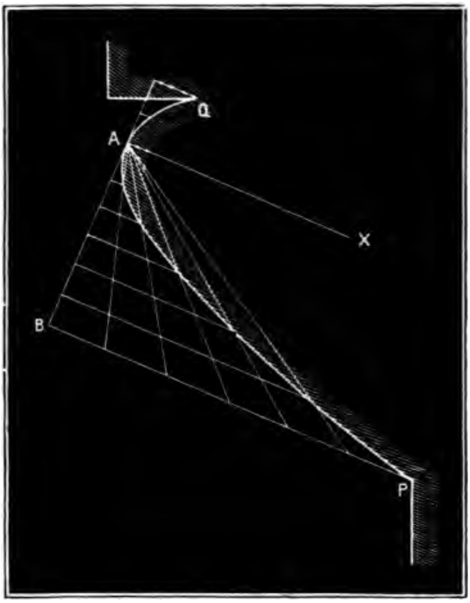


150. *To approximate to the contour of the parabola by arcs of circles.*—Find any number of points, *a*, *b*, &c. (fig. 121) on the curve by one of the methods described above (**142**). Draw the normal at *a* (**146**), cutting the axis at the point *p*, and from *p* as a centre, with *p* A or *p a* as radius, describe the arc A *a*.

Draw the normal at *b*, meeting the normal *a p* in the point *q*, and from *q* as a centre, with *q a* or *q b* as radius, describe the arc *a b*. Proceeding in this way, we can draw a number of arcs of circles gradually increasing in radius, which will approximate to the form of a parabola; and the larger the number of points taken, the nearer will the resulting curve approach to the contour of a parabola.

151. *Application of the parabola to mouldings, architectural ornaments, and other purposes.*—The parabola can be made available to form the outline of a classical ovolo, as Q A P (fig. 122), A X being the axis. Drawing A B at right angles to the axis, and B P parallel thereto, we can find any number of points on the curve by the previous method (**142**).

Fig. 122.



The parabola can also be used for a variety of ornamental purposes where curved lines are required, as in a leaf (fig. 123), of which C D is the axis, D T the tangent at D perpendicular to C D. Then D A C, D A′ C are two similar arcs of the parabola, having A X, A′ X′ for the axes making equal angles with C D, and D T for their common tangent. Other forms of leaf can be obtained in the same way, as shown in figs. 124, 125.

Fig. 123.

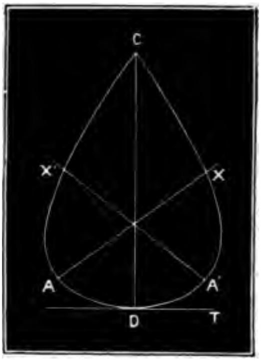
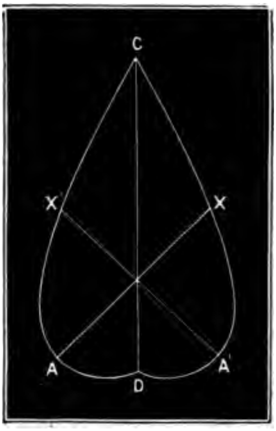
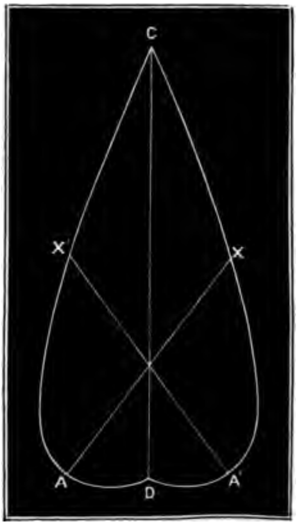


Fig. 124.



If the axis of a parabola is placed vertical so as to have the vertex A (fig. 117) for its highest point, this curve will represent the path which a projectile would take when discharged at any angle to the horizon less than a right-angle, the resistance of the air being entirely ignored. The line of discharge is a tangent to the parabola, and the greatest range is obtained on a horizontal plane when that line makes half a right-angle with the horizon.

Fig. 125.

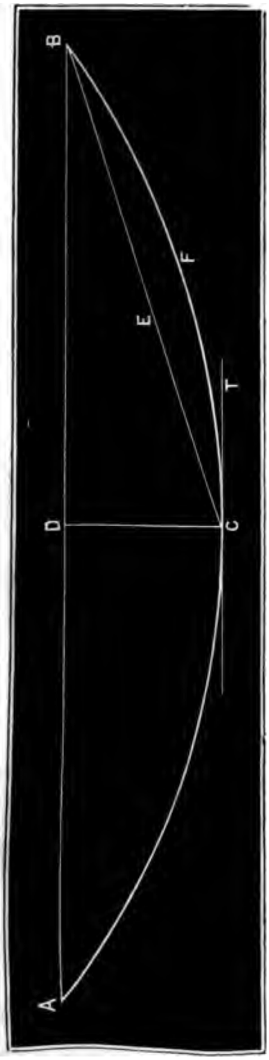


If the axis of the parabola is placed vertical, with the vertex A (fig. 117) for its lowest point, the curve will be that of the chain of a suspension bridge in which the vertical load is uniformly distributed along a horizontal roadway, forming a tangent to the curve at its vertex.

THE CATENARY.

152. DEFINITION.—When a chain or flexible cord is suspended from two points, A and B (fig. 126), in the same horizontal line, the curve which it takes is called a *catenary*. This curve has its greatest curvature at the lowest or middle point C, and the tangent C T is parallel to A B.

Fig. 126.



153. *To apply the catenary to form the entasis of a column.*—Bisect A B (fig. 126) in D, and draw D C at right-angles to A B. Let D C represent the difference between the top and bottom radii of the column, B D its vertical height. Take A D B, equal to twice B D, in a horizontal position on a vertical board; and let a chain be suspended from the points A and B so as to hang down a distance equal to D C from A D B. Then by marking the outline of the chain C F D on the board, the entasis or curvature of the column is obtained.

154. *To apply the catenary to form the entasis of a spire.*—Let A B (fig. 126) represent the slant side of the spire if straight, or the chord of the proposed curve; D C the amount of deviation from the straight line which is required, and which is always very small in comparison with the length of A B. Place A B horizontal upon a vertical board, and allow a chain to hang vertically from A and B to the distance D C, and the outline of the curvature required can be obtained as before.

CHAPTER IV.

CURVES OF FLEXURE.

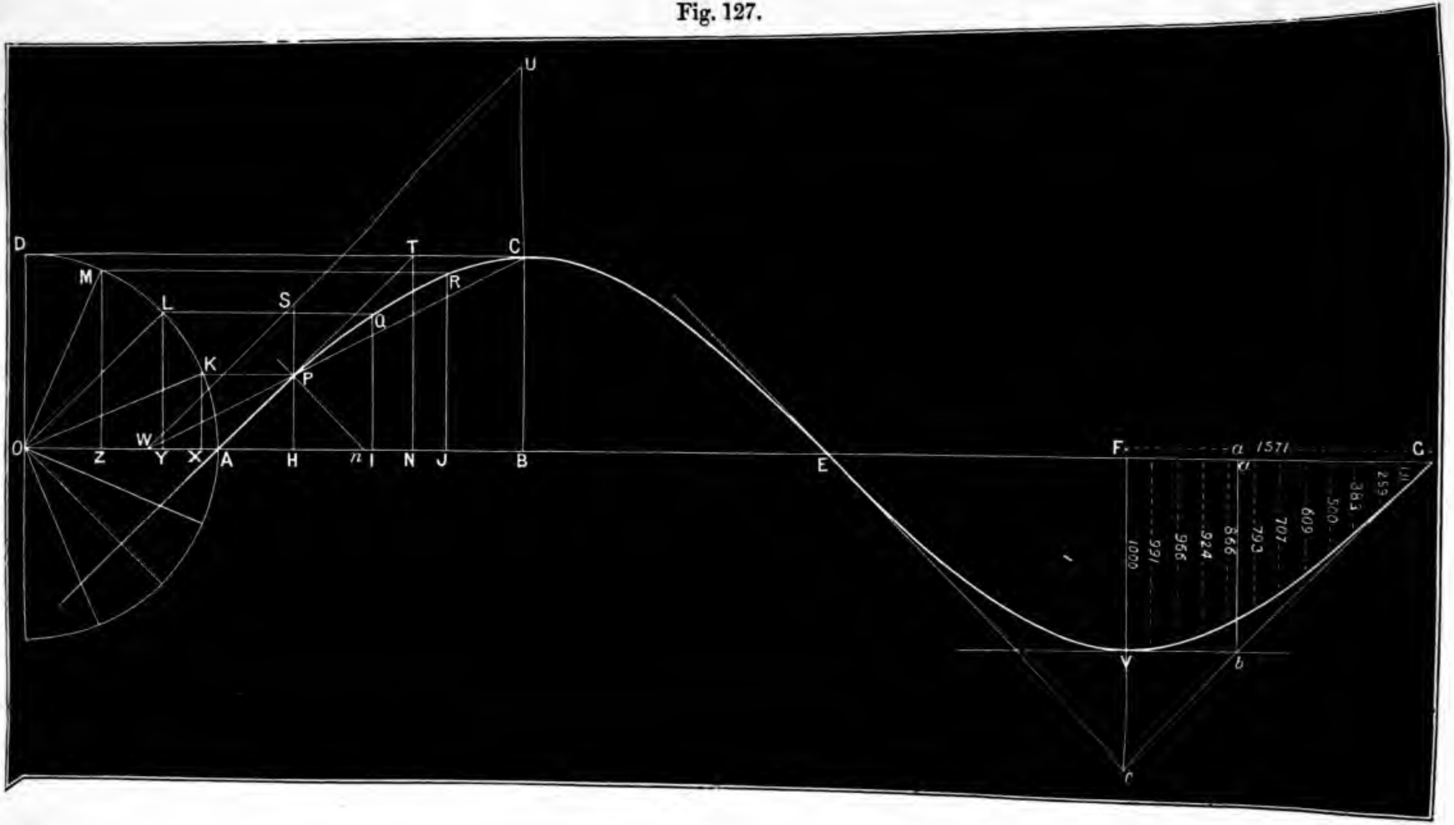
155. DEFINITIONS.—When a curve, after bending in one direction, makes a bend in an opposite direction, like the letter S, it is called a *curve of flexure*; and the point at which the change of direction takes place is called the *point of contrary-flexure.* In all true curves of flexure the curvature decreases as we approach the point of flexure, or the length of the *radius of curvature* (**101**) increases and becomes infinite at that point, the *centre of curvature* passing over to the opposite side and its distance gradually decreasing as we recede from the point of flexure. The *Harmonic-curve*, or *curve of sines*, consists of a series of equal undulations or waves, and is the curve in which a musical string vibrates when sounded. It is called the *curve of sines* from the fact of its ordinates being proportional to the trigonometrical *sines* of the abscissæ.

The *Lemniscate* is a curve having the form of a figure ∞, the point of contrary-flexure being where the curve crosses itself.

THE HARMONIC-CURVE.

156. *To find points on the contour of the harmonic-curve.* —Draw a horizontal line or axis O A B (fig. 127), and draw O D at right angles to O A. Take O D equal to the given height B C, to which the curve is to rise above the axis, and

Fig. 127.



D
M
L
K
O
W
Z
Y
X
A
S
P
H
n
Q
I
T
N
J
R
C
B
U
E
F.
a
1571
G.
a
131
259
383
500
609
707
793
866
924
966
991
1000
b
c

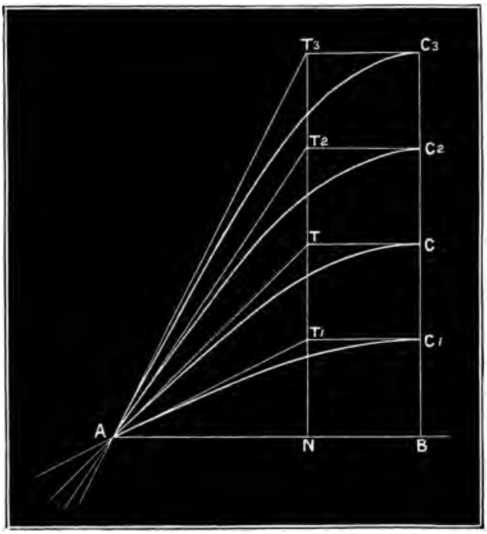
from O as a centre, with O D as radius, describe a circle cutting the axis at A. Take A B equal to the development into a straight line of the quadrant A D; make B E, E F, F G, each equal to A B. Then A G represents the development of the whole circle. Divide the quadrant A D into any number of equal parts at K, L, &c.; and also A B into the same number of equal parts at H, I, &c. Draw verticals at H, I, &c., cutting horizontals at K, L, &c., in the points P, Q, &c., then these will be upon the required curve; and for every point on the quadrant of the circle we can find a corresponding point on the curve. The horizontal D C will be a tangent to the curve at C, and the part of the curve C E is an exact repetition of C A, so also is E V and V G; so that, having drawn the part A C, we can obtain any number of undulations.

The curvature is greatest at the vertex C, and decreases towards A, at which point the radius of curvature becomes of infinite length, A being a point of contrary-flexure; so also are the points E, G, and any other point where the curve cuts the axis.

157. *To find points on the harmonic-curve by a scale of parts.*—Take F G (fig. 127) as an axis, and draw F V at right angles thereto. Measure F V equal to 1000 on any scale, and F G equal to 1571 on the same scale. Divide F G into twelve equal parts, and draw the ordinates; measuring their lengths by the same scale, according to the numbers marked on the figure; then eleven points will be found upon the curve between the axis and the vertex.

Any variety of the harmonic-curve can be produced by measuring the abscissæ on one scale, and the ordinates on some other scale. Let A B (fig. 128) represent the length of a quadrant of the generating circle, draw B C at right-angles to A B, and equal to its radius. Then A C is the curve, having its abscissæ and ordinates measured on the same scale as in fig. 127; the curve A C has the scale of the ordinates half that of the abscissæ; the curve A C_2 has the scales in the ratio of 3 to 2; the curve A C_3 has the scale of the ordinates double that of the abscissæ.

Fig. 128.



158. *To draw the tangent at any point of the harmonic-curve.*—The tangent at A (fig. 127), the point of contrary-flexure, is drawn by taking A N equal to B C, drawing N T vertical cutting the horizontal through C in the point T; then T A is the tangent at A.

If we draw the horizontals through C_1, C_2, C_3 (fig. 128), meeting N T produced in T_1, T_2, T_3; then T_1 A, T_2 A, T_3 A, are the tangents to the curves A C_1, A C_2, A C_3, at the point A.

The tangent at the vertex C is always parallel to the axis A B.

The tangent at any other point of the curve can be drawn by first drawing the normal (**159**) at that point, and then drawing a perpendicular to the normal at the given point.

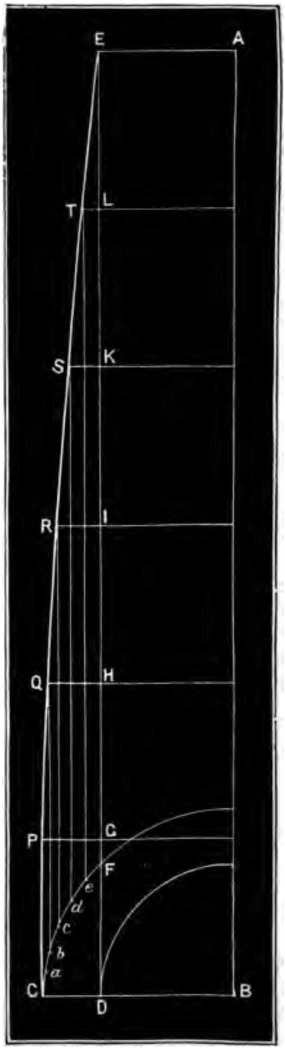
159. *To draw the normal at any given point of the harmonic-curve.*—Let P (fig. 127) be the given point on the curve, HP the ordinate perpendicular to AB. On BA take BJ equal to AH, and draw the ordinate JR, meeting the curve at R. Produce BC to U, making CU equal to JR; draw CPW cutting the axis in the point W, and join WU. Produce HP to meet WU at S, and take H*n* on the axis equal to PS; then P*n* is the normal at the point P; H*n*, called the *subnormal*, being a fourth proportional (**16**) to BC, JR, and HP.

The normal can be drawn in the same way in any variety of the curve (fig. 128), the subnormal being the fourth proportional in every case to BC, JR, and HP; BC being taken the same length in *all* varieties of the curve.

160. *To set out the harmonic-curve as a railway curve.*—Let it be required to join the points E and G (fig. 127) by a curve, the tangents at those points meeting at *e*. Join EG, and bisect EG in F, draw the perpendicular F*e*. Let FG represent 1571 on any scale, and take G*a*, representing 1000, on the same scale. Draw *ab* perpendicular to F*a*, meeting the tangent G*e* in *b*. Draw *b*V parallel to FG. Divide FV into 1000 equal parts, and set up the twelve equidistant ordinates, as figured along FG, to the same scale as that on which FV represents 1000; then twelve points will be determined on the harmonic curve. By repeating the process, the curve from V to E can be set out in like manner.

Where an S curve is required from A to G, the tangents at those points being parallel; divide AG into four equal parts, AB, BE, EF, FG. Draw F*e* at right-angles to FG, and meeting the tangent at G in the point *e*; *e*E will be the tangent at E. Then proceed to set out the curve G V, as above described, and the remaining parts, V E, E C, C A, will be similar to V G, and can be set out in the same manner.

Fig. 129.



By using this curve for railways instead of circular arcs, the sudden change in the direction of the train, which occurs in passing from a straight line to a circle, is avoided, and greater safety insured.

161. *To apply the harmonic-curve to form the entasis of a column.*—Let C B, A E (fig. 129), be the radii at bottom and top of the shaft, B A the axis of the column, B C and A E at right-angles to A B. Take B D equal to A E, then C D is the difference between the radii at bottom and top. Draw D E parallel to B A, and divide it into any number of equal parts at G, H, &c. From B as a centre, with B C as radius, describe a circle cutting D E in the point F. Divide the arc C F into as many equal parts as D E is divided into, by the points *a*, *b*, &c. Through *a*, *b*, &c., draw lines parallel to D E, intersecting lines parallel to B C through G, H, &c., in the points P, Q, &c.; these will be points upon the required curve, through which it can be drawn by bending a lath.

162. *To apply the harmonic-curve to form the contour of an ogival arch or pinnacle.*—Let C F (fig. 130) be the given span of the arch. Bisect C F in E, and draw E D at right-angles to C F. Let E D be the given height of the arch at the centre. Bisect C E in B and draw B A at right-angles to C E, and equal to half E D. Draw C K parallel to A B, and take A H to A B in the ratio of 1000 to 1571. Draw H K parallel to C E and cutting C K in K; then K A is the tangent at the point of contrary-flexure A. Taking C B to represent 1000 on any scale, we can find twelve points on C A, as in V G (fig. 127), by measuring ordinates to the same scale. The curve from A to D is similar to A C, and the curve D F is a repetition of D A C. The joints of the voussoirs of the arch must be in the direction of the normals to the curve (**158**).

Fig. 130.

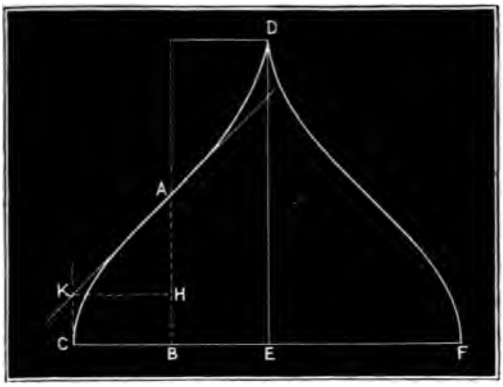
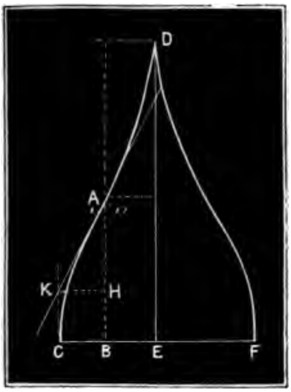


Fig. 131.



The outline of an ogival pinnacle (fig. 131) can be formed in the same way.

163. *To apply the harmonic-curve to mouldings and other architectural ornaments.*—The use of continuous mathematical curves for the contour of mouldings, as the *ogee* and *cima*, would produce a more pleasing effect than the common practice of drawing them by arcs of circles.

Fig. 132.

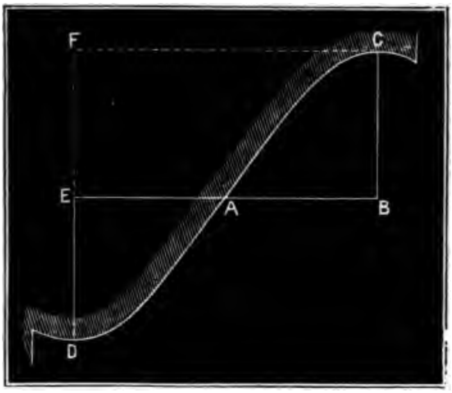
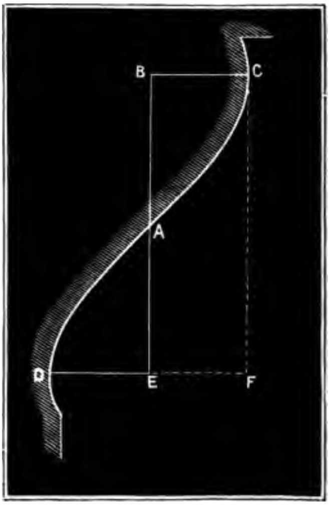


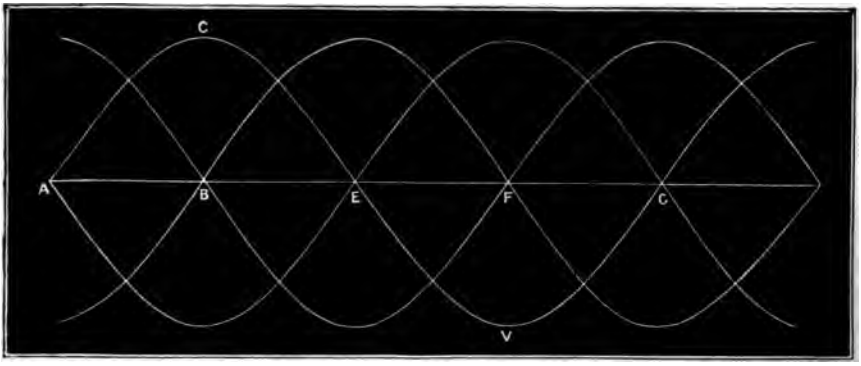
Fig. 133.



Let the points C and D be given (figs. 132, 133); draw C F, D F, at right-angles to each other, and bisect D F in E. Draw E B parallel to C F, B C parallel to D F, and bisect B E in A; then A is the point of contrary-flexure, E A B the axis, D and C the vertices of the curve, which can be drawn in the manner above described (**156**).

The harmonic-curve can also be employed to form an interlacing ornament (fig. 134), A G being the axis, C and V the vertices, A, B, E, F, G, the points of contrary-flexure.

Fig. 134.



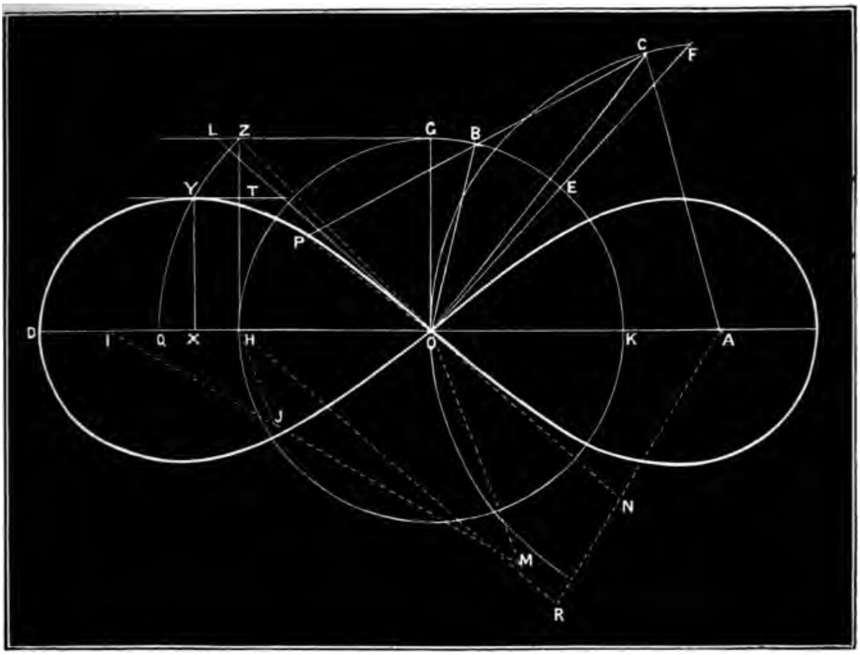
The Lemniscate.

164. *To draw a lemniscate by continuous motion.*—Let A C and O B (fig. 135) be two rods or slips of card revolving about the fixed points A and O, the length A C being capable of alteration. Let another rod or slip, P B C, of length equal to twice O B, be attached by one end to a pivot C at the end of the rod A C, and also at its middle point to a pivot B at the end of the rod O B, so as to move freely about those points, as the rods O B, A C, revolve about O and A. When O B and A C coincide with the axis O A, the point B will be at D, O D being equal to twice O B. Now let the rods revolve about O and A, and a pencil at P will trace out the curve D Y O. When P reaches the axis again at O, the points B and C will be at E and F, in one straight line with O.

The curve from D to O is one-fourth of the whole lemniscate, the other three parts of which are exactly equal and similar to D Y O. Also the point O is a point of contrary-flexure, and the curvature increases from O towards D, where it is greatest.

165. *To find points on the contour of a lemniscate.*—Draw an axis D O A (fig. 135), and from any point O therein as a centre, and any radius O B, describe a circle H B K. Take O A on the axis greater than O K, and from A as a centre, with A O as radius, describe the circle O C F. From O as a centre, with O D equal to twice O H as radius, describe an arc cutting the last circle in F; draw O F cutting the first circle in E.

Fig. 135.



Take any point B on the first circle, between E and H, and from B as a centre, with B O as radius, draw an arc cutting the second circle at C; draw C B P, and make B P equal to B C or B O; then P is a point on the required lemniscate. Proceeding in this way, and taking other points on the first circle, we can find any number of points upon the curve between O and D, through which the lemniscate can be drawn by hand. The other portions of the curve, being only a repetition of the part D Y O, can be drawn from it.

166. *To find the algebraical equation to the lemniscate.*—Since B P (fig. 135) is always equal to B C and B O, the angle P O C is always a right angle (**55**).

Let O A$=a$, O A$=b$, O P$=r$, the radius-vector, and the angle P O D$=\theta$. Then O C$=2\,b\,.\sin\theta$, $OP^2=PC^2-OC^2$, or $r^2=4\,a^2-4\,b^2.\sin^2\theta$. If we call $a=1$, then we have for the equation to the curve,

$$r=2\sqrt{1-b^2\sin^2\theta}.$$

Any varieties of the curve can be obtained by changing the length of b, or O A, as compared with that of a, or O B.

167. *To draw a lemniscate of given dimensions.*—Let the length O D, and the height X Y, be given, Y being the point at which the curve is at its greatest distance from the axis O D. Bisect O D in H, and draw H Z at right-angles to O H, and equal thereto. Draw O Z, and from O as a centre, with O Z as radius, describe the arc Z Y Q, cutting the axis at Q. Then this arc passes through the point of greatest distance, Y, from the axis in every variety of the curve, so that by taking H T equal to the given width or height of the curve, and drawing T Y parallel to the axis, and cutting the arc at Y, we find Y the required point at which the curve has the greatest distance from the axis. The line T Y is also the tangent to the curve at Y. O H is the radius of the smaller circle, and the radius O A of the larger circle is found by taking O H to represent 1000 on any scale, and O A will be the quotient of 1,000,000 divided by the given height X Y, measured to the same scale. For example, let X Y be given equal to one-third of O D, or two-thirds of O H, then by the above rule we find that O A is one and a half times the length of O H.

The length of A O for any given values of X Y and O H can also be found geometrically, since O A is a third proportional (**15**) to X Y and O H. Take H I on the axis equal to X Y, and draw I M making an acute angle with I O; make I J equal to O H; join H J, and draw O M parallel thereto; then J M is the length of O A required.

Having determined the radii of the two generating circles, the curve can be drawn as before described (**164**, **165**).

If O X and X Y only are given, we can find the radius of the smaller generating circle in the following manner. Draw O G at right-angles to O X, and bisect the right-angle G O X by the line O Z. From O as a centre, with O Y as radius, describe a circle cutting O Z in the point Z. Drop the perpendicular Z H upon the axis O H, and O H will be the required radius of the smaller circle; from which D is found by taking H D equal to H O.

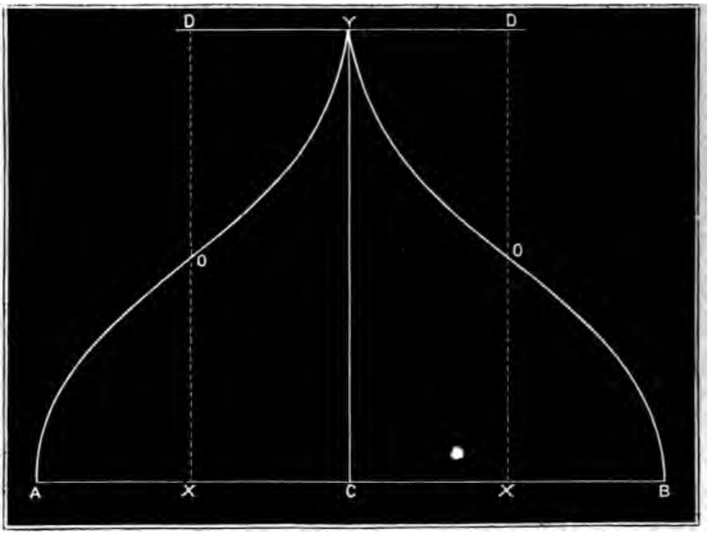
When both the radii, O H and O A, of the generating circles are given, the length of X Y can be obtained by finding a third proportional to O A and O H. Or, take O H as 1000 on any scale, and from O as a centre with a radius 1414 on the same scale, draw a circle; divide 1,000,000 by the length O A (on the same scale), and take H T equal to the quotient; draw T Y cutting the last drawn circle at Y, then Y X is the greatest distance of the curve from the axis.

168. *To draw a tangent to the lemniscate.*—The tangent at O (fig. 135) where the curve cuts the axis, and has a point of contrary-flexure, is found as follows. Draw H Z at right angles to H O and equal thereto. Draw Z L parallel to O H, and from O as a centre, with O A as radius, draw an arc cutting Z L in the point L; then L O is the tangent at O.

The tangent at Y, where the curve is at its greatest distance from the axis, is parallel to O D; and the tangent at D is perpendicular to O D.

169. *To apply the lemniscate to form an ogival arch.*—Let A B (fig. 136) be the given span, C Y the height of the arch, the point C bisecting A B. Bisect A C, B C, in the points X, and draw X O D parallel to C Y, and D Y parallel to A B. Bisect X D in O, and draw the lemniscate Y O (**167**) having D O for its axis, D Y for its greatest distance from the axis, and O for its point of contrary-flexure. Repeat the curve Y O in A O, and Y O A will form one side

Fig. 136.

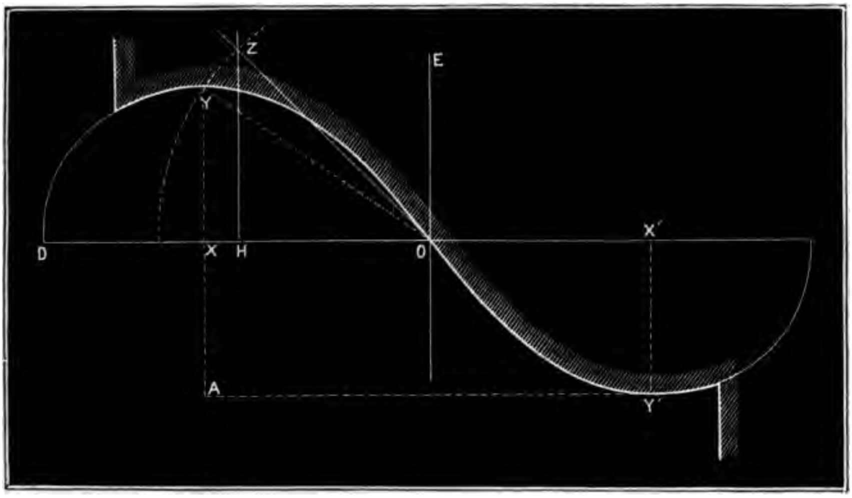


of the required arch, having its tangent at A perpendicular to A B. The tangent at O should be found (**168**) before drawing the curve.

The curve in the figure has the radii of the generating circles in the ratio of 4 to 5.

170. *To apply the lemniscate to mouldings and other architectural ornament.*—Mouldings of contrary-flexure, as shown on figs. 137, 138, 139, can be drawn when the

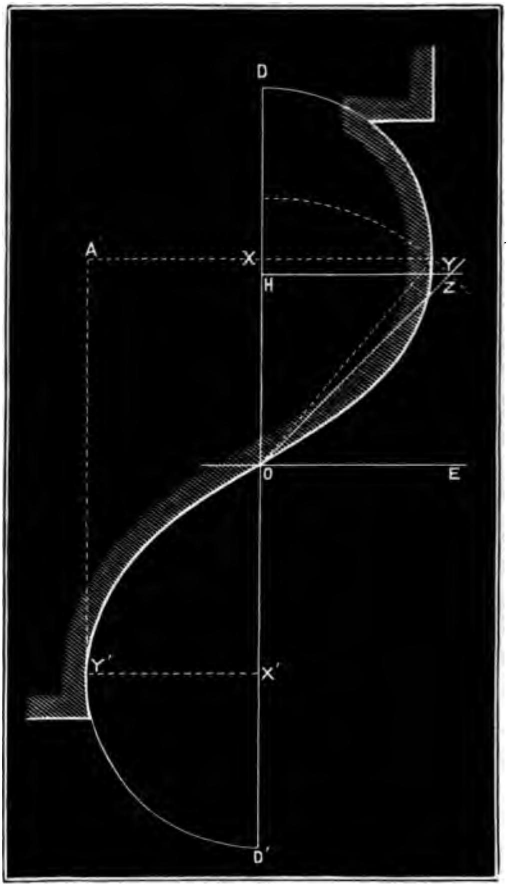
Fig. 137.



points Y, Y′ are given, and the vertical or horizontal axis O D.

Draw Y′ A parallel to O D, and Y X A, Y′ X′, perpendicular thereto. Bisect X X′ in O, which will be the point of contrary-flexure of the curve. Draw O E perpendicular to O X, and bisect the right-angle E O X by the line O Z. From O as a centre, with O Y as radius, describe a circle cutting O Z in Z. Drop the perpendicular Z H upon O X, and O H will be the radius of the smaller generating circle. Make H D equal to O H, and D is the point where the curve cuts the axis. We can now find the length of the other radius in the method described above (**167**), and

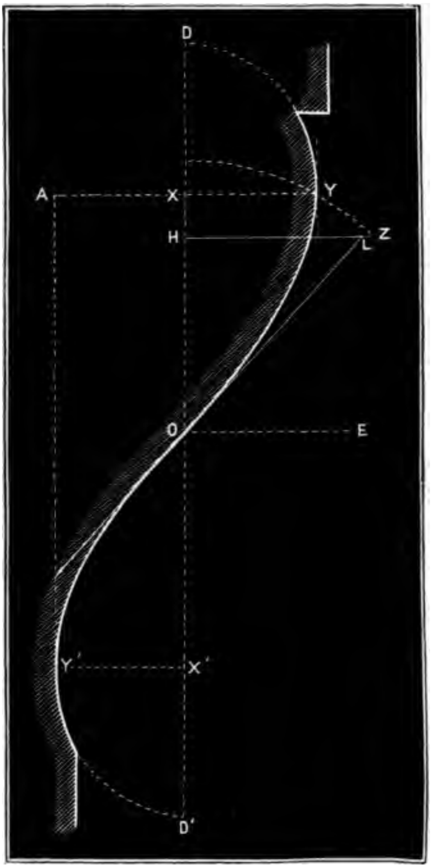
Fig. 138.



draw the curve D Y O. The other portion, O Y′ D′ is a repetition of the portion O Y D. In fig. 137 the radii of the generating circles are in the ratio of 4 to 5, X Y being two-fifths of O D; in fig. 138 they are in the ratio of 8 to 9, X Y being four-ninths of O D; and in figs. 139 and 135 the radii are as 2 to 3, the height X Y being one-third of O D.

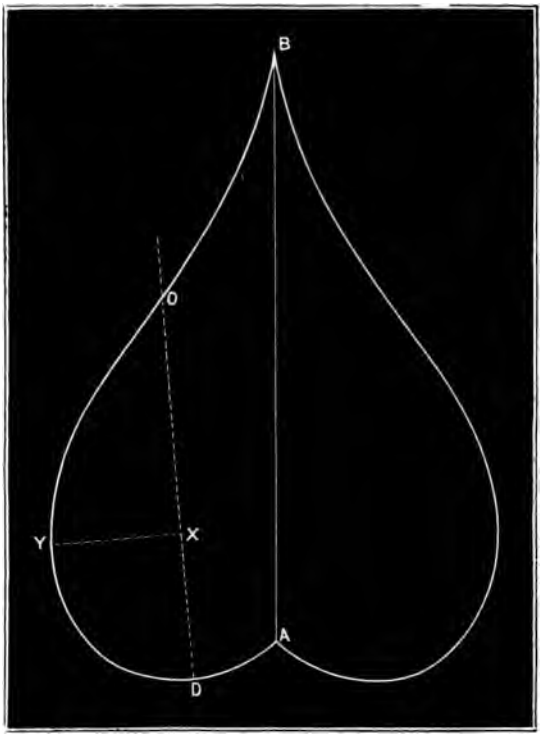
This curve may be also made to form the contour of a leaf (fig. 140), in which O is the point of contrary-flexure, O D the axis, X Y the greatest distance from the axis. In

Fig. 139.



the example here given, the same curve as in fig. 135 is used.

Fig. 140.



There are many other curves having points of contrary-flexure, such as the *Trochoid* and *Companion to the Cycloid*, which are treated upon in Chapter VI.

CHAPTER V.

SPIRALS.

171. DEFINITIONS.—When a point is made to describe a series of convolutions about a fixed point called the *pole*, so that its distance therefrom is continually varying, the curve which it marks out is called a *spiral.* Any line drawn from the pole to the revolving point is called a *radius-vector* to the curve or the radius which the moving point carries round with it. It is usual to consider the radius-vector as starting from some straight line or *axis*, which passes through the pole, and to which it returns after every successive convolution.

The simplest form of spiral is that which is obtained when a thread is unwound from a circular cylinder, the spiral marked out by the end of the thread being called the *Involute of the circle* (fig. 141), in which the tangents of the circle are normals to the spiral.

If a rod is made to revolve round a pole from a fixed axis, and a point starting from the pole moves along the rod so that its distance from the pole is always proportional to the angle which the rod has revolved over from the axis, the curve which the point will mark out is called the *spiral of Archimedes* (fig. 142). This spiral is one of a large family of spirals in which the length of the radius-vector varies according to some *power* of the angle it has revolved over from the fixed axis. Thus we can have a spiral in which the length of the radius-vector is proportional to the *square* or *second-power* of the angle of revolution (figs. 143, 144); or one in which it is proportional to the *cube* or *third-power* of that angle (figs. 145, 146, 147, 148).

When the length of the radius-vector, starting from a fixed axis, *diminishes* in proportion as its angular distance from that axis *increases*, the curve marked out by the moving-point is called the *reciprocal-spiral* (figs. 149, 150), the radius-vector in this case being said to vary *inversely* as the angle of revolution.

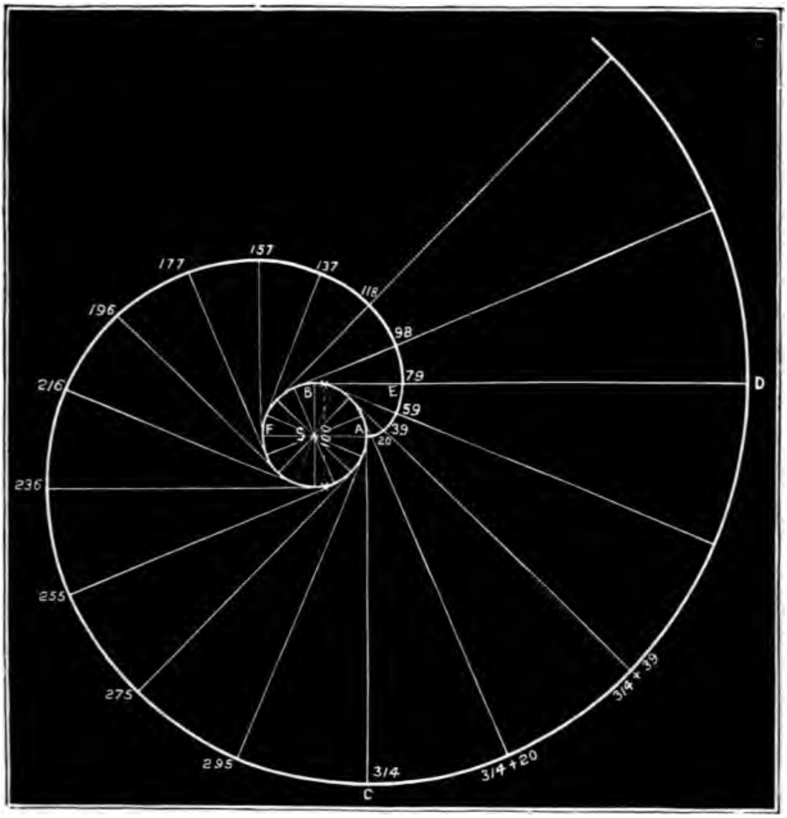
If the length of the radius-vector varies *inversely* as the *square-root* of its angular distance from the axis, the curve marked out is called the *lituus* (fig. 151); and a variety of this spiral can be obtained by making the radius-vector vary *inversely* as the *cube-root* of the angle revolved through (fig. 153).

If the length of the radius-vector is increased in *geometrical ratio*, while the angle of revolution increases in *arithmetical ratio*, the spiral obtained is called the *equiangular* or *logarithmic* spiral (figs. 154, 156, 158). It is called *equiangular* because the tangent at every point in the same spiral makes everywhere the same angle with the radius-vector; and it is termed *logarithmic* because the angle of revolution is proportional to the *logarithm* of the length of the radius-vector. This spiral can be drawn in infinite variety, and has geometrically neither beginning nor ending, as it consists of an endless series of convolutions, all of which are exactly *similar* in the *same* curve, and only differing in actual size or scale.

THE INVOLUTE OF THE CIRCLE.

172. *To draw the involute by continuous motion.*—Let S be the pole, S A the axis (fig. 141) of the proposed spiral.

Fig. 141.



Take a circular cylinder of radius S A, S being its centre, and wrap a thread round it. Attach a pencil to the outer end, A, of the thread, and unwind the thread from the cylinder, keeping it tight by the pencil; then the spiral A E C D will be marked out. If we take A C perpendicular to A S, when the pencil arrives at C the quantity of thread unwound will be equal to the circumference of the cylinder; and since A C is the tangent to the circle at A, the tangent to the spiral at C is parallel to A S, being always at right-angles to the tangent of the circle; and A C is the normal to the spiral at C.

Let it be required to draw a spiral of a given size and given number of convolutions; if A C is given for the first convolution, we must find the diameter A F of the generating circle or cylinder by taking A F to A C in the ratio of 100 to 314, or of 7 to 22. If A C is given for two convolutions, then we must take A F to A C as 100 to 628, or twice 314; if A C is given for three convolutions, take A F to A C as 100 to three times 314, or 942; and so on, according to the number of convolutions required.

173. *To find points upon the involute.*—Divide the generating-circle A B F (fig. 141) into four quadrants, and each quadrant into four equal angles. Draw the tangents to the circle (**48**) at each of the 16 points into which its circumference is divided. Let the diameter of the circle A F represent 100 on any scale; then to find the corresponding points on the spiral, measure along the tangents to the circle by the same scale the distances shown by the numbers on the figure from the points where the tangents touch the circle, beginning from the point A, where the spiral first starts from the circle. In this way 16 points will be found on the first convolution from A to C. To find points on the second convolution, produce the above-named tangents indefinitely, and measure a distance of 314 on the same scale as before, from each point on the first convolution; these will give 16 points on the second convolution. To find points on the third convolution, measure

314 from each point on the second, and so on for any number that may be required.

A larger number of points can be found upon the spiral if desired. Suppose it is required to find 32 points on each convolution; bisect each of the 16 angles into which the circle was divided above, and draw the tangents at the extremities of the radii. The length of the tangent at the end of the first radius after leaving A will be 314 divided by 32; that of the second, twice that of the first; of the third, three times that of the first, and so on for all the thirty-two points round the circle. For the points on the second convolution we have only to proceed as before, by adding 314 to the length of each normal to the first convolution, and in the same way for the third convolution.

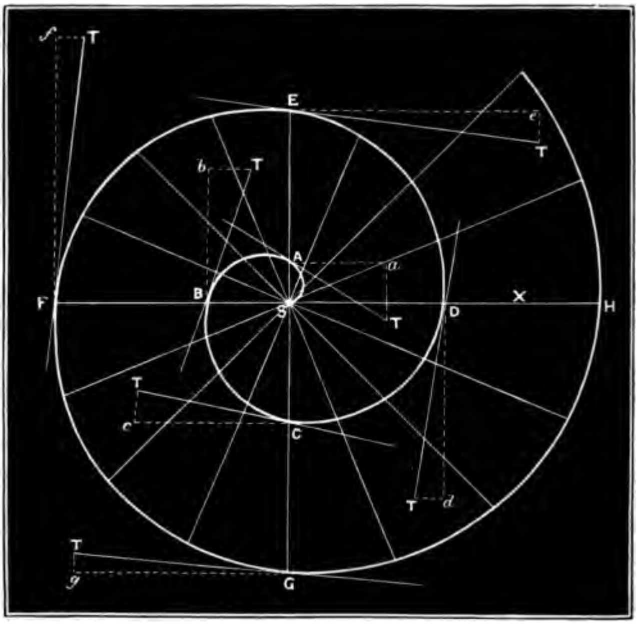
When a cogged-rack is *driven* by a pinion-wheel, the outline of the cogs is the involute of the circle which forms the 'pitch line' of the 'driver.'

THE SPIRAL OF ARCHIMEDES.

174. *To draw a spiral whose distance from the pole is directly proportional to the angle made by the radius-vector with the fixed axis.*—Let F S H (fig. 142) be the fixed axis, S the fixed point, or pole. The spiral can be drawn by continuous motion if a rod is made to revolve about S, and at the same time a pencil moves along the rod at a rate proportional to the velocity of revolution.

To describe the spiral by finding points on its contour, draw E S G at right-angles to S H, and divide each right-angle into four equal angles. Let D be the point where the spiral is to cut the axis after one convolution; on S E take S A, equal to one-fourth of S D, on S F take S B, equal to half of S D, and on S G take S C, equal to three-fourths of S D. So also for the four angles into which the first quadrant is divided, the length of the radius-vector drawn from S is one-fourth of S A at the first quarter of the right-angle, one-half S A at the second quarter, and three-fourths S A for the third quarter. In the other quadrants the length of each radius-vector increases in the same way by one-fourth of S A for every quarter of a right-angle.

Fig. 142.



If we bisect each of the 16 angles, the increase of the radius-vector at each successive angle will be one-eighth of S A.

The distance apart, D H, of any two convolutions of this spiral, as measured along the radius-vector, is everywhere the same, being equal to the length S D.

175. *To draw the tangent at various points of the foregoing spiral.*—Let ϕ be the angle between the tangent and radius at any point of the curve after the radius-vector has passed over an amount of revolution from the axis which is represented in magnitude by θ (taken in *circular* measure); then it is found by the differential calculus that

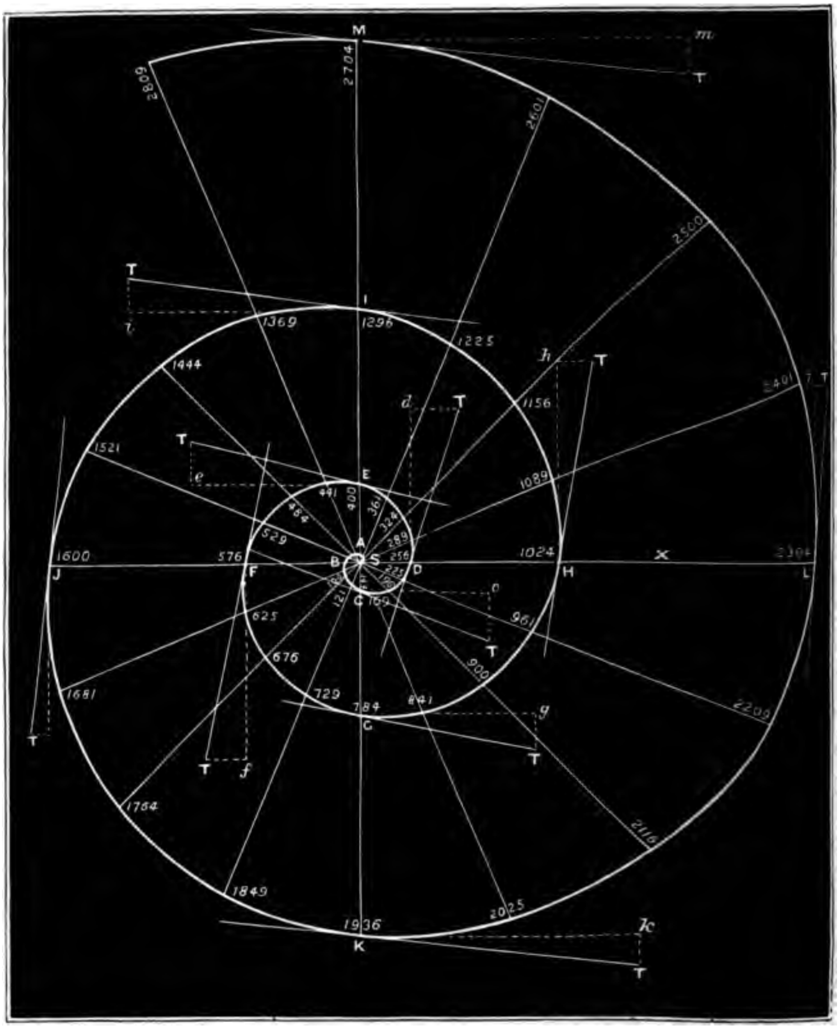
$$\tan\phi=\theta.$$

From this formula it appears that the angle which the tangent makes with the radius-vector is very acute for any point near the pole S, and increases rapidly as its distance therefrom is increased. Thus at A, the tangent makes an angle T A S of 57·52° with the radius-vector; at B the angle T B S is 72·67°; at C the angle T C S is 78°; at D the angle T D S is 81°; at E the angle T E S is 82·75°; at F the angle T F S is 84°; at G the angle T G S is 84·8°; and at H the angle is 85·45°. And as the number of convolutions increases, so the curvature of the spiral gets nearer and nearer to that of a circle having S for its centre.

To draw the tangent at A, make A *a* perpendicular to S A, and equal on any scale to 157; draw *a* T at right angles to A *a*, and equal to 100 on the same scale, then T A is the tangent at A. Proceeding in the same way at the other points, at B make B *b* equal to 314, *b* T to 100; then T B is the tangent at B. At C, C T the tangent is drawn by taking C *c* to *c* T as 471 to 100. At D, take D *d* to *d* T as 628 to 100. At E, take E *e* to *e* T as 785 to 100. At F, take F *f* to T *f* as 942 to 100. At G, take G *g* to T *g*, as 11 to 1.

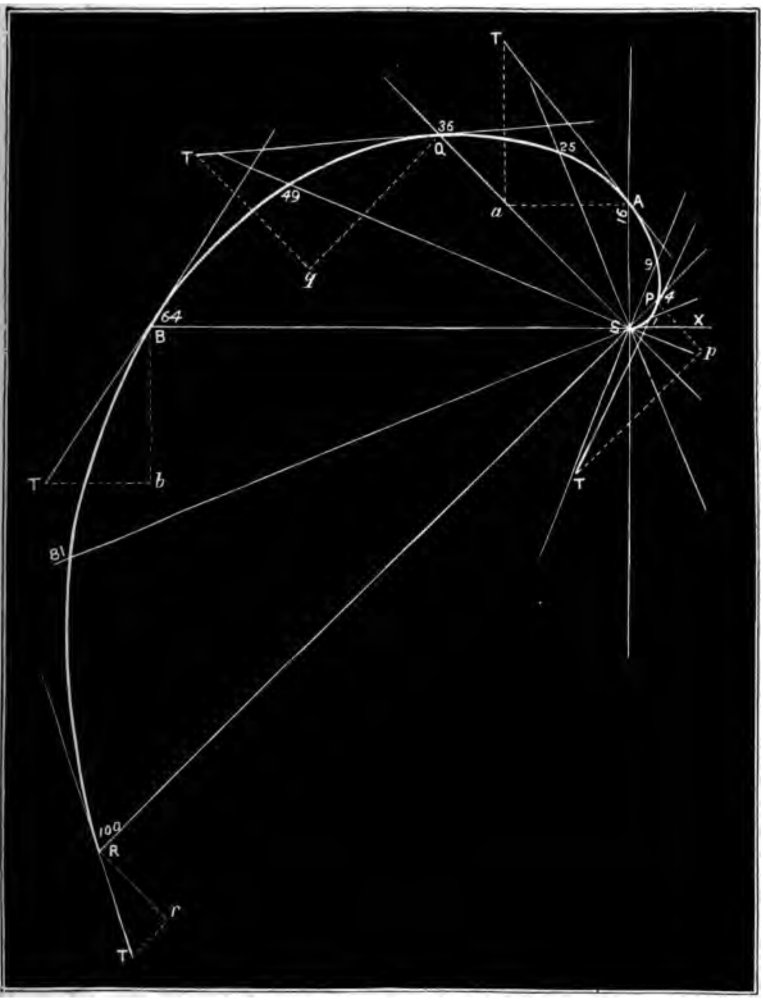
176. *To find points on the spiral whose distance from the pole is proportional to the square of the angle made by the radius-vector with the fixed axis.*—Let S (figs. 143, 144) be the pole, S X the fixed axis. Draw K S M (fig. 143) perpendicular to S X, and divide each quadrant into four equal angles. Measure along these lines from S the *squares* of successive integers beginning with unity, as given in the Table (**181**), on any convenient scale; then sixteen points will be obtained on each convolution of the spiral. To

Fig. 143



save reference to the Table, the numbers are marked on the figures themselves, indicating the distance of the various

Fig. 144.



points from the pole. The spiral can be drawn by hand through the points thus found.

177. *To draw the tangents at various points of the foregoing spiral.*—If we call ϕ the angle which the tangent at any point of the curve makes with the radius-vector drawn to that point, and θ the amount of revolution described by the radius-vector from the axis (taken in *circular* measure), it is found by the differential calculus that

$$\tan \phi = \tfrac{1}{2} \theta.$$

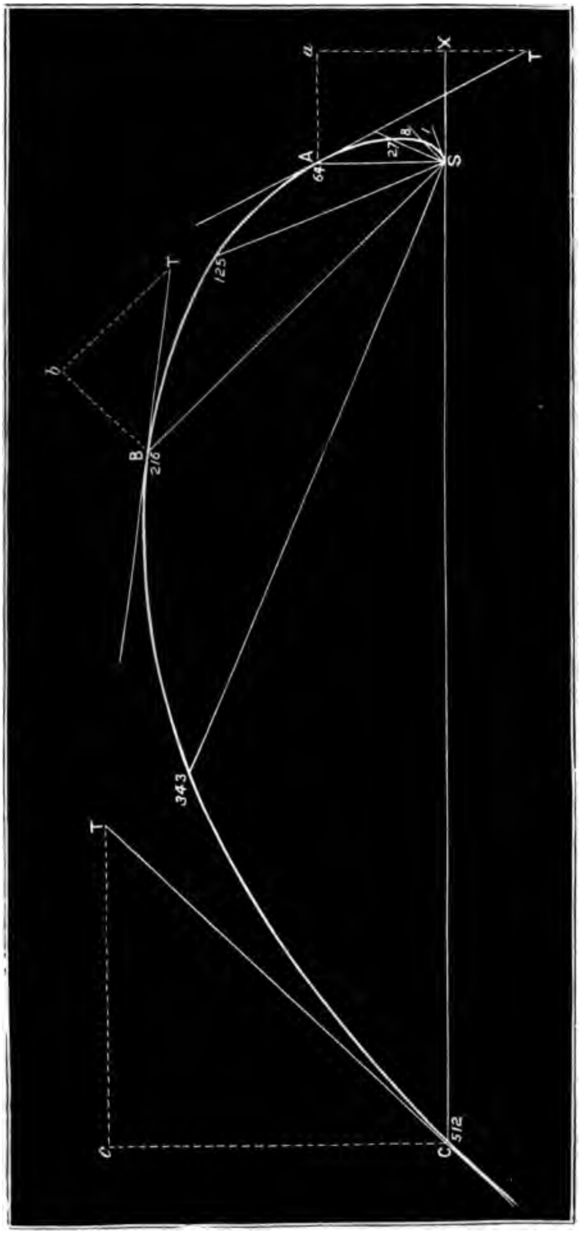
From this formula it appears that the angle between the tangent and radius is very acute for any point near the pole S, but increases rapidly as the number of convolutions is increased.

Let S P (fig. 144) make half a right angle with S X, its length being 4; draw P p perpendicular to P S, and measure P p, equal to 39 on any scale, make p T at right-angles to p P and equal to 100 on the same scale, then T P is the tangent at P. Proceed in the same way at the other points, taking A a to T a as 78 to 100, and T A is the tangent at A. At Q, take Q q to T q as 118 to 100, and T Q is the tangent; at B, take B b to T b as 157 to 100, and T B is the tangent; at R, take R r to T r as 196 to 100, and T R is the tangent. At C (fig. 143), take C c to T c as 236 to 100, and T C is the tangent; at D, take D d to T d as 314 to 100; at E, take E e to T e as 393 to 100; at F, take F f to T f as 471 to 100. At G, take G g to T g as 55 to 10, and T G is the tangent; at H, take H h to T h as 63 to 10; at I, take I i to T i as 71 to 10; at J, take J j to T j as 79 to 10; at K, take K k to T k as 86 to 10; at L, take L l to T l as 94 to 10; at M, take M m to T m as 102 to 10, and T M is the tangent.

Before drawing any spiral by hand, it is essential to draw as many tangents as possible, since they give the *direction* in which the curve is moving at the several points.

178. *To find points on the spiral whose distance from the pole is proportional to the cube of the angle made by the* *radius vector with the fixed axis.*—Let S (figs. 145, 146,

Fig. 145.



147, 148) be the pole, S X the fixed axis. Through S draw a line at right-angles to S X, and divide each quadrant into four equal angles. Measure from S along these lines the *cubes* of all successive integers, beginning with unity, as given in the Table (**181**), on any convenient scale; sixteen points on each convolution will be thus obtained. The numbers marked on the figures themselves give the distances of the several points from the pole, and the spiral can be drawn by hand through the points thus found.

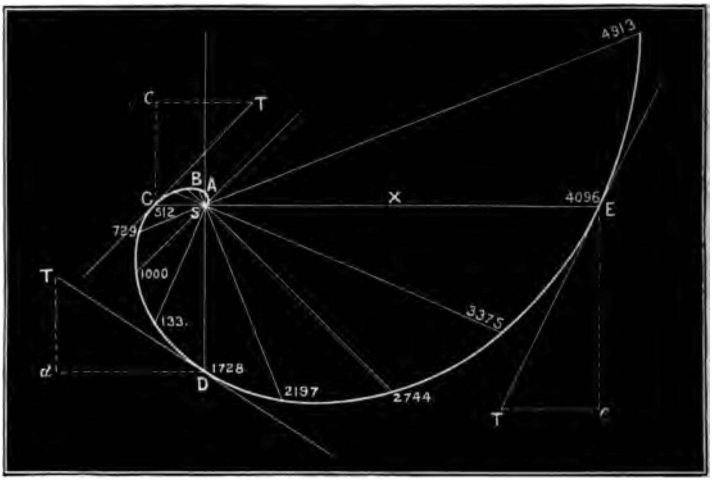


Fig. 146.

Figure 145 shows the spiral for the first half-convolution starting from the pole.

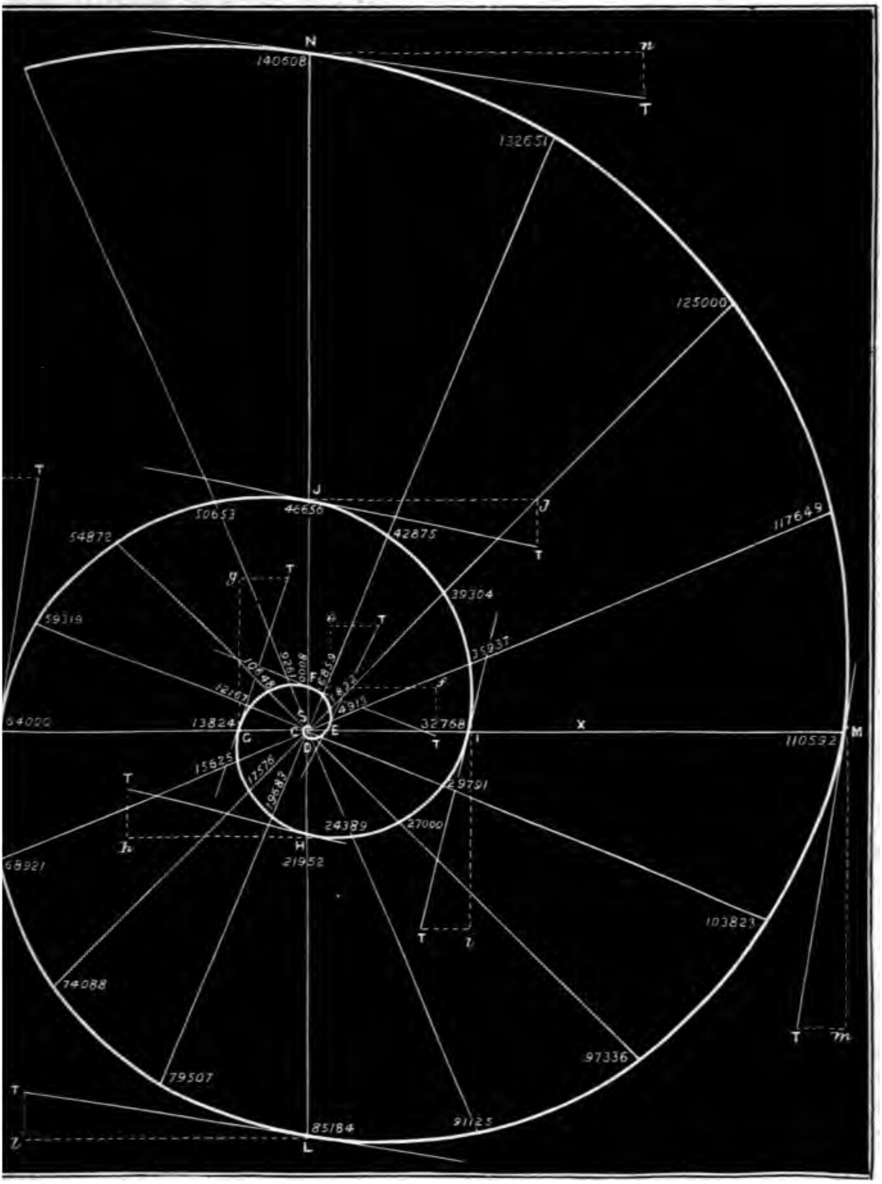
Figure 146 gives the first whole convolution of the spiral from the pole on a scale one-twentieth of that used in the former figure.

Figure 147 shows three and a quarter convolutions drawn to a scale one-twentieth of that used in figure 146.

Figure 148 gives six and a quarter convolutions from the pole, drawn to a scale one-tenth of that used in drawing figure 147.

179. *To draw the tangents at various points of the fore-*

Fig. 147.



going spiral.—Let ϕ be the angle which the tangent and radius-vector make with each other at any given point on the curve, and let θ be the angle (in *circular* measure) which the radius-vector at that point has revolved over from the axis; then it is found by analysis that

$$\tan \phi = \tfrac{1}{3}\theta.$$

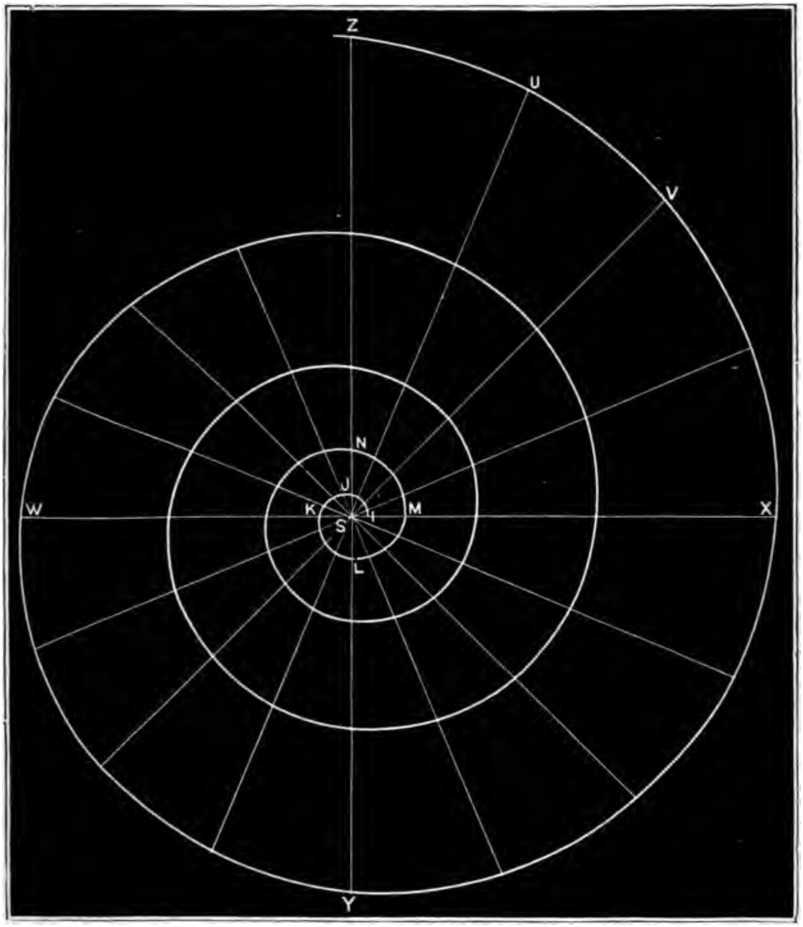
The angle ϕ is acute and less than half a right-angle in the first half-convolution from the pole, and after that becomes greater than half a right-angle, increasing as the curve proceeds, and getting nearer and nearer to a right-angle.

To draw the tangent at A (fig. 145), take A*a* perpendicular to S A, and equal in length to 53 on any convenient scale, erect *a* T at right-angles to *a* A and equal to 100 on the same scale, then T A is the tangent at A. Proceeding in the same manner we find the tangent at B by taking B*b* to T*b* as 79 to 100; at C, take C*c* to T*c* as 105 to 100, and T C is the tangent.

To draw the tangent at D (fig. 146), take D*d* to T*d* as 157 to 100; at E, take E*e* to T*e* as 209 to 100; at F (fig. 147), take F*f* to T*f* as 262 to 100; at G, take G*g* to T*g* as 314 to 100; at H, take H*h* to T*h* as 373 to 100; at I, take I*i* to T*i* as 419 to 100; at J, take J*j* to T*j* as 471 to 100; at K, take K*k* to T*k* as 524 to 100; at L, take L*l* to T*l* as 587 to 100; at M, take M*m* to T*m* as 628 to 100; and at N, take N*n* to T*n* as 694 to 100; then T N will be the tangent at N.

150. *To apply the above spiral to form a volute of given dimensions.*—Let Y Z (fig. 148) be the given height of the volute. Divide Y Z at S, so that S Z shall be to S Y in the ratio of the cube of 100 to the cube of 92 (see Table of cubes (**181**)), or nearly as 100 to 78. Then draw W S X at right-angles to Y Z, and divide each quadrant into four equal angles by lines drawn through the pole S; take S X

Fig. 148.



to S W as the cubes of 96 and 98, or nearly as 88½ to 68, and measure on the other lines the lengths as given in the table of cubes, starting from S Z as the cube of 100, then S U the cube of 99, S V the cube of 98, and so on for as many convolutions as may be required.

If it is desired to finish the spiral at the centre by a circular *eye* at any point as N, draw the normal or perpendicular to the tangent at N, and also the normal at M; then describe the eye taking the point of the intersection of these two normals for its centre.

If it is desired to draw an approximation to the spiral by means of arcs of circles joining the several points found in the foregoing manner, draw the normals at those points perpendicular to the tangents, and the point where two consecutive normals intersect will be the centre from which the arc lying between them is to be struck.

181. *Table of the Squares and Cubes of Integers.*

Integers	Squares	Cubes	Integers	Squares	Cubes
1	1	1	31	961	29791
2	4	8	32	1024	32768
3	9	27	33	1089	35937
4	16	64	34	1156	39304
5	25	125	35	1225	42875
6	36	216	36	1296	46656
7	49	343	37	1369	50653
8	64	512	38	1444	54872
9	81	729	39	1521	59319
10	100	1000	40	1600	64000
11	121	1331	41	1681	68921
12	144	1728	42	1764	74088
13	169	2197	43	1849	79507
14	196	2744	44	1936	85184
15	225	3375	45	2025	91125
16	256	4096	46	2116	97336
17	289	4913	47	2209	103823
18	324	5832	48	2304	110592
19	361	6859	49	2401	117649
20	400	8000	50	2500	125000
21	441	9261	51	2601	132651
22	484	10648	52	2704	140608
23	529	12167	53	2809	148877
24	576	13824	54	2916	157464
25	625	15625	55	3025	166375
26	676	17576	56	3136	175616
27	729	19683	57	3249	185193
28	784	21952	58	3364	195112
29	841	24389	59	3481	205379
30	900	27000	60	3600	216000

Table of the Squares and Cubes of Integers (continued).

Integers	Squares	Cubes	Integers	Squares	Cubes
61	3721	226981	81	6561	531441
62	3844	238328	82	6724	551368
63	3969	250047	83	6889	571787
64	4096	262144	84	7056	592704
65	4225	274625	85	7225	614125
66	4356	287496	86	7396	636056
67	4489	300763	87	7569	658503
68	4624	314432	88	7744	681472
69	4761	328509	89	7921	704969
70	4900	343000	90	8100	729000
71	5041	357911	91	8281	753571
72	5184	373248	92	8464	778688
73	5329	389017	93	8649	804357
74	5476	405224	94	8836	830584
75	5625	421875	95	9025	857375
76	5776	438976	96	9204	884736
77	5929	456533	97	9409	912673
78	6084	474552	98	9604	941232
79	6241	493039	99	9801	970299
80	6400	512000	100	10000	1000000

THE RECIPROCAL SPIRAL.

182. *To find points on the spiral in which the length of the radius-vector varies inversely as the angle of revolution from a fixed axis.*—Let S (fig. 149) be the pole, S X the fixed axis. Draw C S A at right-angles to S X, and let the length S A represent 1000 on any scale. Draw A B parallel to S X, then A B will be an asymptote (**124**) to the curve. Draw S K, making an angle of 57·3° with the axis, and from S as centre, with S A as radius, describe an arc cutting S K in K, then K is a point on the curve. Bisect the right-angle A S X by the line S I, making S I equal to 1273 on the above scale; then I is a point on the curve.

Draw S Q, bisecting the angle K S X, and equal in length to twice S K, or 2000 on the scale, then Q is a point on the curve. Draw S R, bisecting the angle I S X, taking S R equal to twice S I, or 2546 on the scale, then R is a point on the curve, and the curve gets nearer and nearer to the asymptote A B as the angle between the radius-vector and the axis diminishes.

Draw S L, bisecting the angle A S I, and take S L equal to one-third of S R, or to two-thirds of S I, or 849 on the scale; then L is a point on the curve.

On S A, take S M equal to half S I, or 637 on scale, and M is a point on the curve. So also S D will be half of S M, S C one-third of S M, S E one-fourth of S M, and so on.

To find other points on the spiral, divide each quadrant into four equal angles, and measure the lengths marked on the figure by the same scale as is used in the other dimensions; and in this way sixteen points on each convolution will be obtained. In figure 150 the spiral portion of the curve is drawn on an enlarged scale three times that to which figure 149 is drawn.

The portion of the curve P Q R (fig. 149) is very flat, but the curvature gradually increases towards M. Thus at I the radius of curvature (**101**) is more than four times the length of the radius-vector S I; at K it is less than three times the radius-vector; and at M it is less than twice the radius-vector. As the curve continues in its spiral course the radius of curvature gets nearer and nearer to the radius-vector in length.

A second spiral shown by the dotted line I′ K′ L′ (fig. 149) can be drawn in the same manner as the first, by taking S A′ and S K′ as 1000 on some other scale than that used for S A and S K; A′ B′ will be the asymptote to this curve, and the other points can be found by scale as before.

Another scale of measurement can be used by dividing each right-angle, as before, into four equal angles by the lines S R, S I, S L, &c., R S X being one-fourth of a right-angle, I S X one-half, K S K three-fourths. Let S R represent unity on any scale, then S I will be half of S R, S L will be one-third of S R, S M will be one-fourth, S D will be one-eighth, S C will be one-twelfth, S E will be one-sixteenth of S R, and so on round the spiral.

183. *To draw tangents at various points on the reciprocal spiral.*—Let ϕ be the angle made by the radius-vector at any point on the curve with the tangent at that point, θ the angle of revolution (in *circular* measure) of the radius from the fixed axis S X (figs. 149, 150), then it is found by analysis that

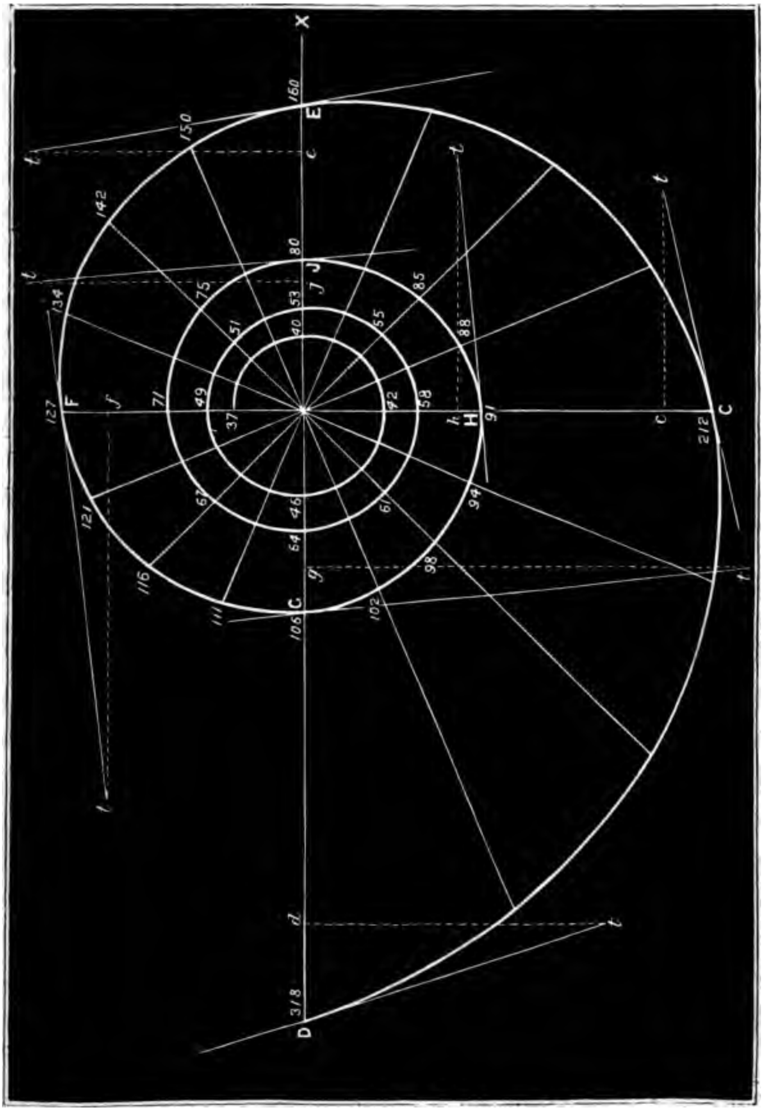
$$\tan \phi = \theta.$$

The tangent K*t* at K (fig. 149) makes half a right-angle with S K, so that to draw K*t* we have only to take K*k* and K*t* at right-angles, and equal to each other. To draw the tangent at M, take M*m* equal to 100 on any scale, draw *m t* a perpendicular at *m*, and take thereon *m t* equal to 157 on the same scale, then *t*M is the tangent at M. At D (fig. 150), take D*d* equal to 100, *dt* equal to 314 perpendicular to D*d*, and *t*D is the tangent at D. Proceeding in the same way at other points, we have at C, C*c* to *ct* as 10 to 47; at E, E*e* to *et* as 100 to 628; at F, F*f* to *ft* as 100 to 786; at G, G*g* to *gt* as 100 to 943; at H, H*h* to *ht* as 1 to 11; at J, J*j* to *jt* as 100 to 1257. The tangent at any other points can be found if required in the same manner by help of the formula given above.

It will be seen that the tangent becomes more and more nearly perpendicular to the radius-vector as the spiral increases its number of convolutions.

By means of the tangents we can draw the curve by hand through the several points found upon its contour, as

Fig. 150.



they give us the direction in which the spiral is moving.

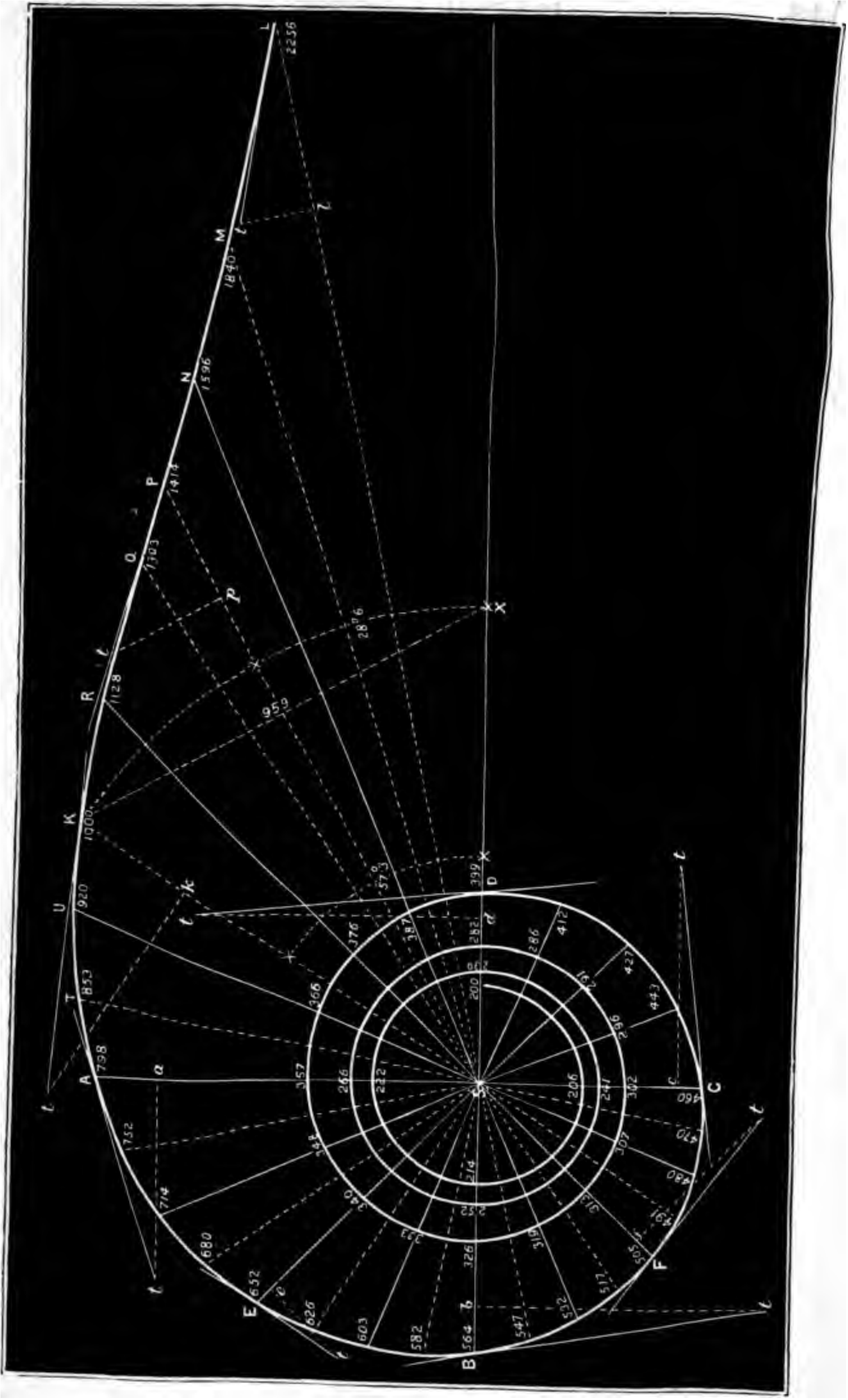
If it is desired to imitate the spiral by means of arcs of circles joining the several points found in the above manner, the normals must be drawn perpendicular to the tangents at those points, and the point of intersection of any two consecutive normals will be the centre from which any one of those arcs must be struck.

THE LITUUS.

184. *To find points on the contour of the spiral in which the radius-vector varies inversely as the square root of the angle of revolution from a fixed axis.*—Draw BSX (fig. 151) as a fixed axis, S being taken as the pole. Measure a length SX equal to 1000 on any scale, and from S as a centre, with SX as radius, describe a circle; from X as a centre, with a radius equal (on the same scale) to 959, draw an arc cutting the circle in K, which will be a point on the required curve. K can also be found by making the angle KSX equal to 57·3°, and measuring SK as 1000. Bisect the line KX by the line SP, taking SP equal to 1414 on the above scale; then P is a point of contrary-flexure on the curve, which now becomes convex towards the axis SX as it approaches thereto, being concave from P towards K. As the angle made by the radius-vector with the axis decreases, so the curve approaches nearer and nearer to the axis, but as the length of the radius increases at the same time, the curve never can reach the axis which is an asymptote (**124**) thereto.

Draw CSA perpendicular to SX, and divide each quadrant into eight equal angles, measuring the distance of each point as shown upon the figure to the same scale as was

Fig. 151.


L
M
1840
l
N
1596
P
Q
p
R
r
K
X
x
U
k
t
D
d
T
A
a
c
C
F
e
E
B
b

used in measuring S K; we thus obtain a sufficient number of points to enable us to draw in the spiral by hand.

The radius of curvature (**101**) is of infinite length at the point of contrary-flexure, P, or the curve becomes flat at that point. From P towards A the curvature increases rapidly, or the length of the radius of curvature diminishes; thus at K the radius of curvature is not quite double the length of the radius-vector S K, at A it is less than one and a half times S A, and at D it is very little more than the length of the radius-vector S D, so that from this point the curvature of the spiral approaches very nearly to a circle.

The line S R bisects the right-angle A S X and measures 1128 on the before-mentioned scale; the line S N bisects the angle R S X and measures 1596; the line S Q bisects the angle R S N and measures 1303; the line S L bisects the angle N S X and measures 2256; the line S M bisects the angle N S L and measures 1840. The line S U bisects the angle A S R, and the line S T bisects the angle A S U.

185. *To draw tangents at various points of the foregoing spiral.*—Let ϕ be the angle between the tangent and radius-vector at any given point on the curve, θ the angle (in *circular* measure) of revolution made by the radius with the fixed axis S X (fig. 151), then it is found by analysis that

$$\tan \phi = 2\,\theta.$$

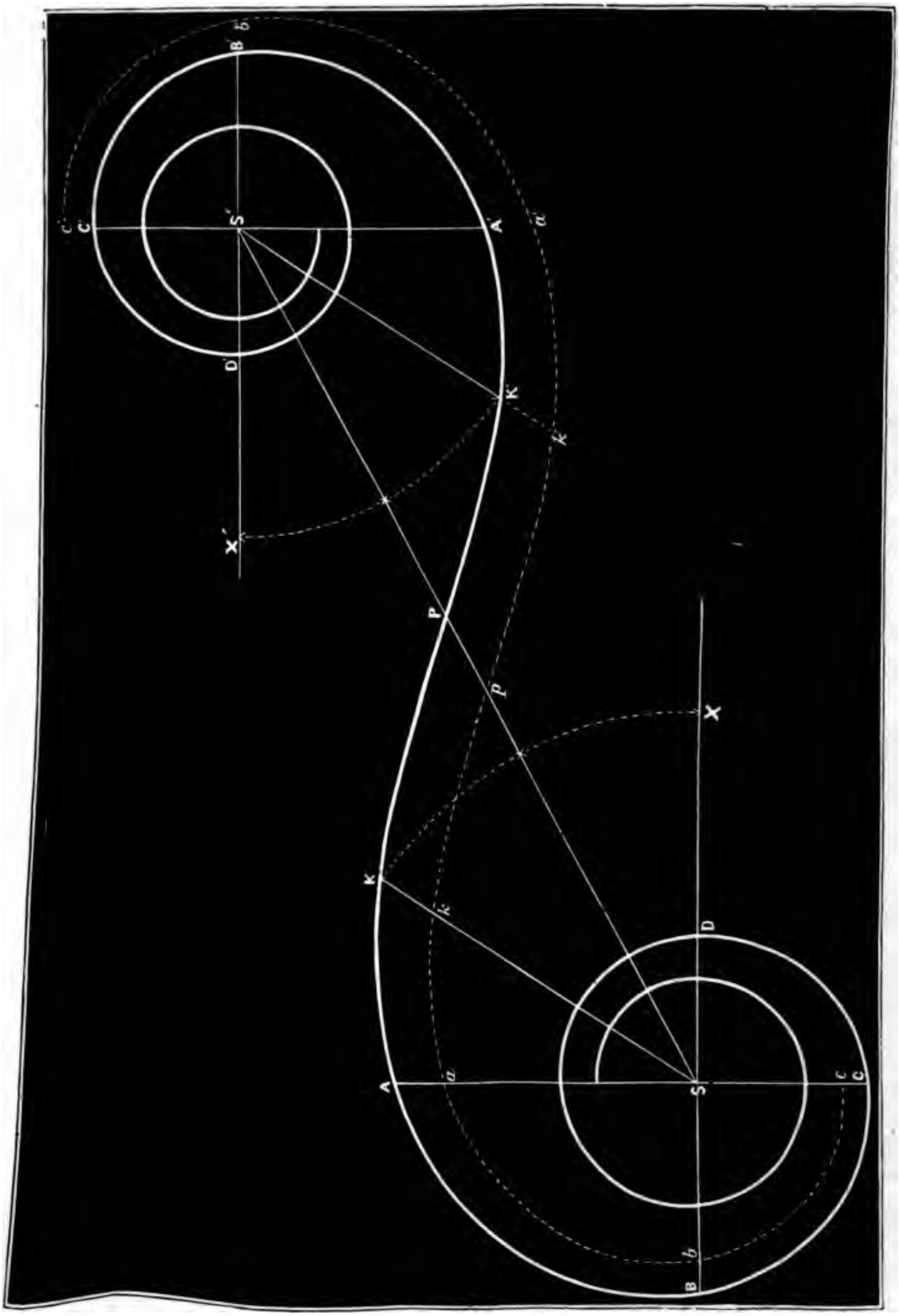
By help of this formula the tangent can be drawn for any given angle of revolution.

At L, take L l, on the line S L, equal to 100 on any scale, draw $l\,t$ perpendicular thereto and measuring 39 on the same scale, then tL is the tangent at L. Proceeding in the same way, we find the tangent at P by taking P*p* equal to *p t*, and *t* P is the tangent;. at K, take *k t* equal to twice K *k*, and *t* K is the tangent; at A, A *a* is to *a t* as 100 to 314: at E, E *e* is to *e t* as 100 to 471; at B, B *b* is to *b t* as 100 to 628; at F, F *f* is to *f t* as 100 to 785; at C, C *c* is to *c t* as 100 to 943; at D, D *d* is to *d t* as 100 to 1257. At the point D the tangent and radius-vector S D make an angle of 85½°, and as we follow the spiral inwards this angle gets nearer and nearer to a right-angle, or the curve approaches towards a circle in its contour.

Having drawn the tangents we can describe the curve itself by hand through the several points; or if it is desired to approximate to the curve by arcs of circles drawn from point to point, the centres of those arcs must be at the intersection of the consecutive normals or perpendiculars to the tangents at those points.

186. *To apply the lituus to form the outline of a console or double volute.*—Draw a horizontal axis B S X (fig. 152), take S X equal to 1000 on any scale, and from S as a centre, with S X as radius, draw the circle X K; from X as a centre, with radius equal to 959 on the same scale, draw an arc cutting the circle in K; or draw K S, making an angle of 57·3° with the axis. Bisect the angle K S X by the line S P S′, and take S P equal to 1414, then P is the point of flexure of the curve. On the same or any other scale measure P S′ equal to 1414, and draw B′ S′ X′ parallel to B S X; take S′ X′ equal to 1000 on the same scale as P S′ is measured upon, and from S′ as a centre, with S′ X′ as radius, describe a circle. Draw S′ K′, making with S′ P an angle equal to the angle P S′ X′, and cutting the circle at K′. P being the point of contrary-flexure in both curves, and the two curves having a common tangent at P,

Fig. 152.



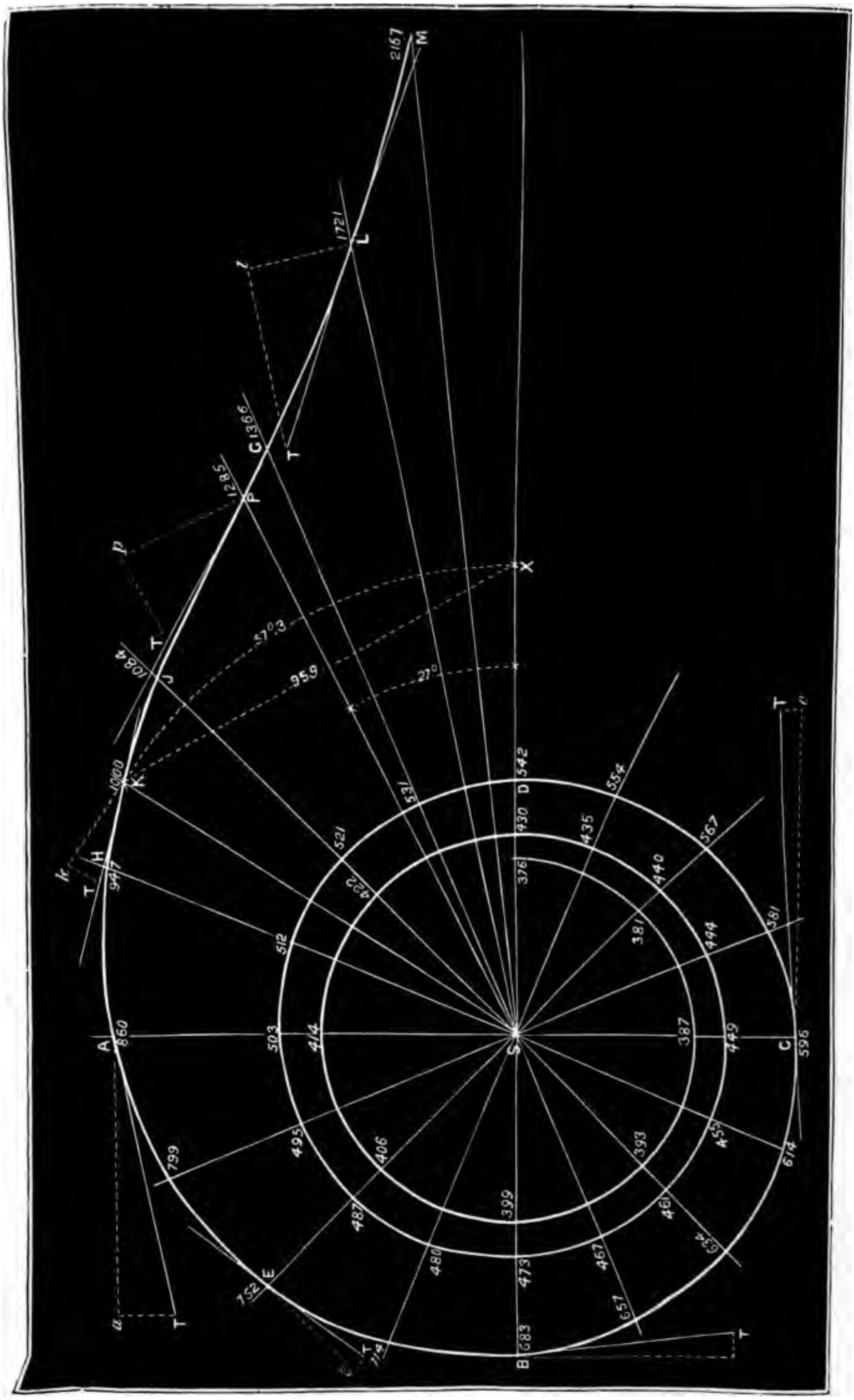
a continuous contour is obtained in this manner, and the two spirals can be drawn by the method above described (**185**).

The dotted line *c b a p a′ c′* indicates another pair of spirals drawn around the first pair, having the same poles S and S′ and the same axes S X and S′ X′, but measured on different scales. In the figure S *p* is taken equal to S′ P, and S′ *p* equal to S P; but any variation can be made in the scales employed.

187. *To find points upon a spiral in which the radius-vector varies inversely as the cube-root of the angle of revolution from a fixed axis.*—Let B S X (fig. 153) be the fixed axis, S the pole, C S A at right-angles to the axis. Take S X equal to 1000 on any scale, and from S as a centre, with S X as radius, describe a circle; from X as a centre, with 959 on the same scale as radius, draw an arc cutting the circle in K, which is a point on the curve; or K will be found by making the angle K S X equal to 57·3°. Draw S P, making with the axis an angle of 27°, and measure S P equal to 1285, then P is the point of contrary-flexure, the curve becoming convex to the axis as it approaches it from the point P; and the axis is an asymptote (**124**) to the curve.

To find other points on the spiral, divide each quadrant into four equal angles, and measure the lengths from S as marked on the figure by the foregoing scale. Thus S J bisects the angle A S X and measures 1084 on the same scale as S K measures 1000; bisect J S X by the line S G, and measure S G equal to 1366. Bisect the angle G S X by the line S L, and measure S L equal to 1721; bisect L S X by S M, and measure S M equal to 2167. The line S H bisects the angle J S A, and is equal to 947.

Fig. 153.



M
1721
L
l
T
C 1366
P
p
T
J
57°.3
27°
X
K
k
H
T
D
A
S
C
E
T
u
T
B
T

This spiral can be made into a double spiral in the same manner as described above (**186**), by joining on another similar curve at the point P, and using either the same or a different scale for the two spirals.

188. *To draw tangents at various points of the foregoing spiral.*—Let ϕ represent the angle between the tangent and radius-vector at any point on the curve, θ the angle (in *circular* measure) which the radius-vector has revolved over from the fixed axis; then it is found by analysis that

$$\tan \phi = 3\,\theta.$$

From this formula we can draw the tangent at any given point on the curve. Thus at L the tangent T L is drawn by making L *l* perpendicular to the radius-vector S L, and equal to 59 on any scale; then take *l* T equal to 100 on the same scale. At P, we find P *p* to T *p* as 1414 to 1000, and T P is the tangent at P; proceeding in the same way at K, we find K *k* to T *k* as 3 to 1; at A, A *a* is to T *a* as 471 to 100; at E, E *e* is to T *e* as 707 to 100; at B, B *b* is to T *b* as 942 to 100; and at C, C *c* is to T *c* as 1414 to 100.

After passing C the tangents become very nearly perpendicular to the radius-vector, and the curvature approaches that of a circle.

Having drawn the tangents the curve can readily be described by hand through the several points; or by drawing normals perpendicular to the tangents at those points, arcs of circles can be described from point to point having their centres at the intersections of consecutive normals.

The Equiangular or Logarithmic Spiral.

189. *To find points on the equiangular spiral, the depth and axis of which are given.*—Let D S C (fig. 154) be the given axis, S the pole, A S B the given depth; the ratio of A S to B S being also given. Find S C a mean-proportional (**17**) between A S and B S, then C is a point where the spiral cuts the axis. Join A C, and draw B D parallel to A C cutting the axis in D, then D is also a point on the curve. Join B C, and draw D E parallel to B C, and E is a point where the spiral cuts S A. Draw E G parallel to

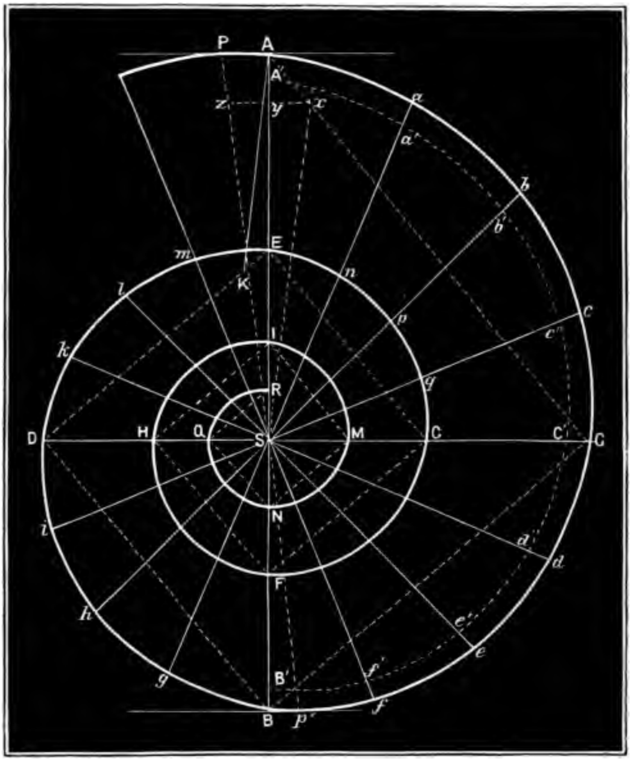


Fig. 154.

A C, G F parallel to B C, F H parallel to B D, H I parallel to D E, and so on for as many convolutions as may be desired, and we obtain four points on each convolution.

To find intermediate points, divide each quadrant into four equal angles by the lines S *a*, S *b*, &c.; then the line S *b* is a mean-proportional between S A and S C; S *a* is a mean-proportional between S A and S *b*; S *c* is a mean-proportional between S *b* and S C; S *e* is a mean-proportional between S C and S B; S *d* is a mean-proportional between S C and S *e*; S *f* is a mean-proportional between S *e* and S B. Having determined eight points on the first half of the convolution, the other points are found from them; for the chord of B *g* is parallel to the chord of A *a*, the chord of *g h* is parallel to the chord of *a b*, and so on all round the spiral. The points where the other convolutions cut the lines drawn from S are found in the same manner by drawing chords parallel to the chords in the first convolution; thus E *n* is parallel to A *a*, *n p* to *a b*, and so forth.

Another method.

Divide the given length A S by the given length B S, and take the common logarithm of the quotient; divide 1·3643 by that quotient, and call the result α; or, in algebraical formula,

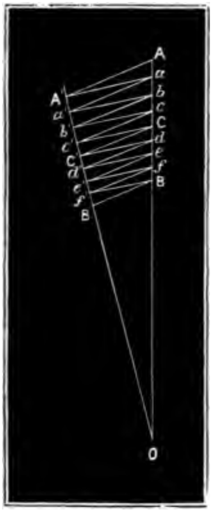
$$\alpha = \frac{1{\cdot}3643}{\log\left(\frac{AS}{BS}\right)}$$

Next divide ·1705 by the value of α found above, and subtract the quotient from the common logarithm of S A; the difference will be the common logarithm of S *a*; or, in algebraical formula,

$$\log(Sa) = \log(SA) - \frac{\cdot 1705}{\alpha}$$

Having thus determined the length of S a, draw two lines O A and O A′, making an acute angle with each other at O (fig. 155), let O A equal S A, O A′ less than O A; take O a equal to S a (fig. 154), determined as above, join A A′, A′ a; then draw $a\,a'$ parallel to A A′, $a'\,b$ parallel to A′ a, $b\,b'$ parallel to A A′, $b'\,c$ parallel to A′ a, and so on; then the lengths O a, O b, O c, &c., thus found on O A will represent the length S a, S b, S c, &c., on the spiral (fig. 154); and in this way all the points on the contour from A to B can be found. Also if we require to draw another spiral similar to the first, as shown by the dotted line A′ a' b' (fig. 154), we have only to take O A′ (fig. 155) equal to S A′, and the lengths O a', O b', O c', &c., will represent S a', S b', S c', &c., on the second curve.

Fig. 155.



190. *To draw tangents and normals at various points on the equiangular spiral.*—Let the lengths S A, S B (fig. 154) be given; then the *trigonometrical* tangent α of the angle between the radius-vector and the *geometrical* tangent at any point of the spiral is found by dividing S A by S B, taking the logarithm of the quotient and dividing the number 1·3643 by the quantity obtained; or, algebraically expressed,

$$\alpha = \frac{1 \cdot 3643}{\log\left(\frac{AS}{BS}\right)}$$

Suppose, for example, that S A = 20, S B = 14, as in figure 154; then we find from this formula that $\alpha = 9$. Upon S A measure S y equal to 9 on any scale, draw $y\,z$ at right-angles, and equal to 1 on the same scale, or to one-ninth of S y; join S z, then S y divided by $y\,z$ represents the trigonometrical tangent (α) of the angle S $z\,y$ or S $x\,y$ ($y\,x$ being taken equal to $y\,z$); therefore S $x\,y$ or S $z\,y$ is the angle which the tangent and radius-vector make with each other at every point of the spiral; and if A K is drawn parallel to S x, A K is the normal or perpendicular to the tangent at A; and the tangent itself can be drawn by simply drawing a line at A perpendicular to A K. If A z is produced to meet the curve at P, then P is the point where the tangent is parallel to the axis S C, and is the highest point on the convolution. The normals at a, b, c, &c., can all be found by drawing lines making an angle equal to K A S with S a, S b, S c, &c., and the tangents at those points will be perpendicular to those normals. If the curve is drawn in by hand the tangents should all be drawn at the several points so as to give the direction of the spiral. If it is desired to imitate the spiral by drawing arcs of circles from point to point, the centres of those circles will be found in the intersection of the normals drawn from any two consecutive points.

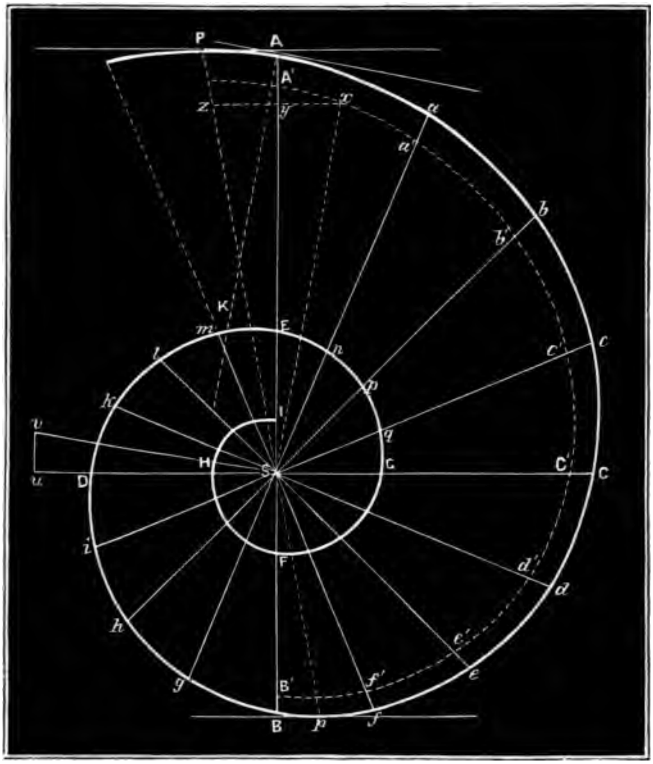
191. *To draw the equiangular spiral when the depth and the angle between the tangent and radius are given.*—Let A B (fig. 156) be the given depth, α the *trigonometrical* tangent of the given angle, the value of which is found from a table of tangents when the angle is given. Then the ratio of A S to B S is found by the following formula:

$$\log\left(\frac{\text{AS}}{\text{BS}}\right) = \frac{1{\cdot}3643}{\alpha}$$

or, divide 1·3643 by the value of α and the quotient is the logarithm of the ratio of A S to B S, the value of which is found from a logarithmic table. We have then to divide A B in the ratio thus obtained and find the pole S. The spiral can now be drawn by one of the methods above described (**189**).

For example, let the given angle be 80°, as in figure 156; the tangent of 80° is given in the tables as 5·67,

Fig. 156.



which is the valne of α in this case. Dividing 1·3643 by 5·67, we get ·2406, which is the logarithm of 1·74, or very nearly $\frac{7}{4}$. Divide the given length A B into eleven equal parts, and measure four of those parts from B towards A, and we obtain the pole S. Draw D S C at right-angles to A B, and we find the point C by taking S C a mean-proportional between A S and B S, either by the geometrical process (**17**), or by extracting the square root of the product of A S and B S, for

$$SC^2 = AS \times BS = 7 \times 4 = 28.$$

This gives us S C = 5·3 very nearly.

The point b is found by taking S b a mean-proportional between S A and S C, or, $Sb = \sqrt{7 \times 5{\cdot}3} = 6{\cdot}1$.

The point a is found by taking S a a mean-proportional between S A and S b, or,

$$Sa = \sqrt{7 \times 6{\cdot}1} = 6{\cdot}53.$$

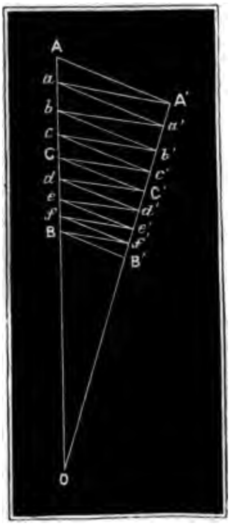
We can also obtain the value of S a by the formula previously given (**189**), namely,

$$\log (Sa) = \log (SA) - \frac{{\cdot}1705}{\alpha} = \log (7) - \frac{{\cdot}1705}{5{\cdot}67}$$
$$= {\cdot}8451 - {\cdot}03 = {\cdot}8151 = \log. (6{\cdot}53)$$

or, S a = 6·53 as found above.

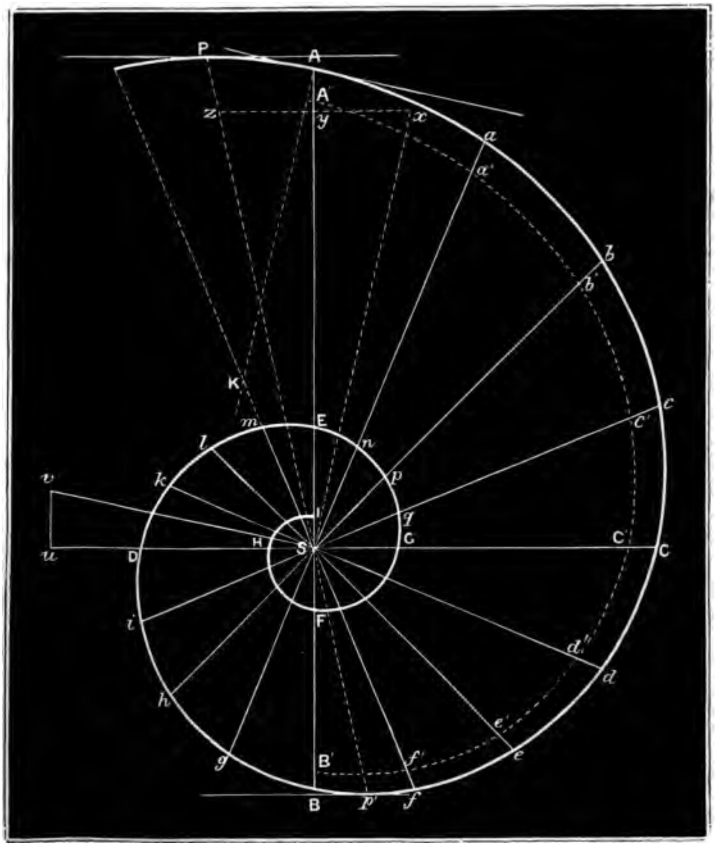
Now draw O A, O A′ (fig. 157), making an acute angle at O, and take O A′ less than O A, as, for example, let O A′ equal S A′ (fig. 156). Take O A equal to S A (fig. 156), and O a equal to S a just found, namely 6·53, the length of S A being 7. Join A A′ (fig. 157) and A′ a; draw $a\,a'$ parallel to A A′, $a'\,b$ parallel to A′ a, $b\,b'$ parallel to A A′, $b'\,c$ parallel to A′ a, and so on; then O a, O b, O c, &c, represent the lengths of S a, S b, S c, &c. (fig. 156). Also O a', O b', &c., represent the lengths S a', S b', &c., in the dotted spiral.

Fig. 157.



On S A (fig. 156) take S y equal to 5·67 on any scale, draw $z\,y\,x$ perpendicular to S y, making $y\,z$ and $y\,x$ each equal to unity on the same scale; then A K the normal at A is parallel to S *x*. Produce S *z* to meet the curve at P, and the tangent at P is parallel to D C. Also produce S D to *u*, and let S *u* be 5·67 on any scale, draw *u v* perpendicular thereto and equal to unity on the same scale; then

Fig. 158.



the tangent at A is parallel to S *v*; it is also at right-angles to the normal A K. S *v u* represents the angle which the tangent and radius-vector make with each other at every point on this spiral.

As another example, let A B (fig. 158) be given as 30 on any scale, A S equal to 20, B S to 10, the ratio of A S to B S being that of 2 to 1; then (**190**) we have

$$\alpha = \frac{1{\cdot}3643}{\log (2)} = \frac{1{\cdot}3643}{{\cdot}30103} = 4{\cdot}53$$

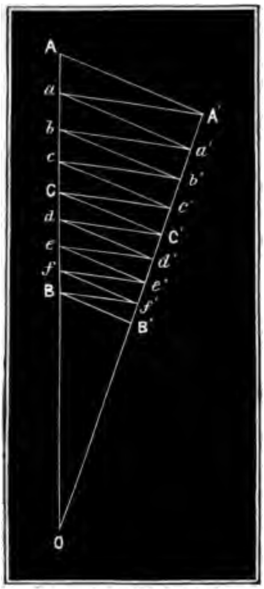
which is found by the tables to be the tangent of 77·55°, and this is the angle between the tangent and radius-vector at every point on the spiral. Take S y equal to 4·53 on any scale, draw $x\,y\,z$ perpendicular thereto, and let $x\,y$ or $y\,z$ be unity on the same scale; then the normal A K at A is parallel to S x; and if the line S z is produced to meet the curve at P, that will be the point where the tangent is parallel to the axis D C. If we take S u equal to S y, or to 4·53 on any scale, and draw $u\,v$ perpendicular and equal to $y\,z$ or to unity on that scale; then the line S v is parallel to the tangent at A. The angle S $v\,u$ or S $x\,y$ represents the angle (77·55°) which the tangent and radius-vector make with each other at all points of the curve. The length of S C is a mean-proportional between 20 and 10, or is the square-root of their product, 200; this gives S C = 14·14; S b is the square-root of 20 × 14·14, or 16·8; S a is the square-root of 20 × 16·8, or 18·34. The value of S a can also be found directly from the formula previously given (**189**);

$$\begin{aligned} \log (\mathrm{S}\,a) &= \log (20) - \frac{{\cdot}1705}{4{\cdot}53} \\ &= 1{\cdot}30103 - {\cdot}0376 \\ &= 1{\cdot}26343 \end{aligned}$$

which is found in the tables to be the logarithm of 18·34.

When S A and S a (fig. 158) are determined, the other points b, c, &c., can be found by drawing O A and O A′ (fig. 159) making an acute-angle, O A being taken equal to S A (fig. 158), and O A′ equal to S A′ and less than S A. Take O a equal to S a, draw A A′, A′ a; then proceed as before to draw successive parallels to those two lines, and O b, O c, &c., will be equal to S b, S c, &c. (fig. 158). Also O a', O b', &c., will be equal to S a', S b', &c., on the dotted spiral.

Fig. 159.



If it is desired to draw the arcs A a, a b, &c., as arcs of circles, the normals must be drawn at those points, making an angle of 77·55° with the radii S A, S a, S b, &c., and then the point where the normals at A and a intersect will be the centre for the arc A a; the point where the normals at a and b meet will be the centre for the arc a b; and so on.

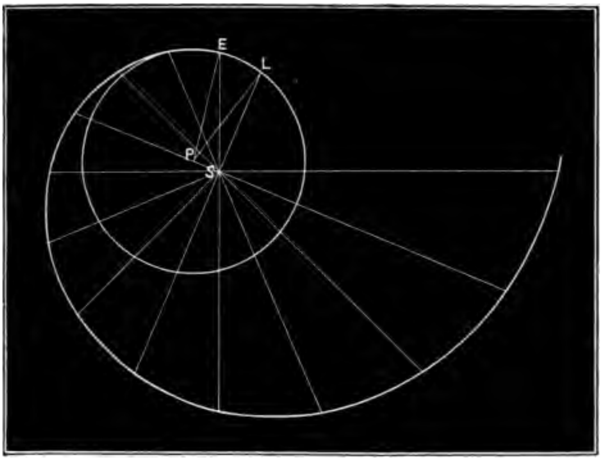
192. *To apply the equiangular-spiral to the formation of volutes and other architectural ornaments.*—In drawing volutes the depth A B (fig. 154, 156, 158) and the ratio of A S to B S are usually given. The volute can therefore be drawn in the manner above described (**189**).

The point A will not, however, be the highest point of the spiral which rises until it reaches P found in the manner before described (**190**), and at this point the tangent is parallel to the axis D C. Also if P S is produced to meet the curve again at p, the tangent at p will be also parallel to D C, and the point p will be the lowest on the spiral.

The spiral given in figure 154 is a very near approximation to the usual form of Ionic volute, having S A to S B as 10 to 7. If another spiral A′ a' b', as shown by the dotted line, is required, as is the case when there is a fillet or moulding on the face, we must proceed as in figure 155, taking O A equal to S A, O A′ to S A′; O *a*, O *b*, &c., equal to S *a*, S *b*, &c., and drawing the parallels we obtain O *a*′, O *b*′, &c., equal to the required lengths of S *a*′, S *b*′, &c., and the spiral can be drawn in as before. In the same way any number of lines consisting of similar spirals can be obtained.

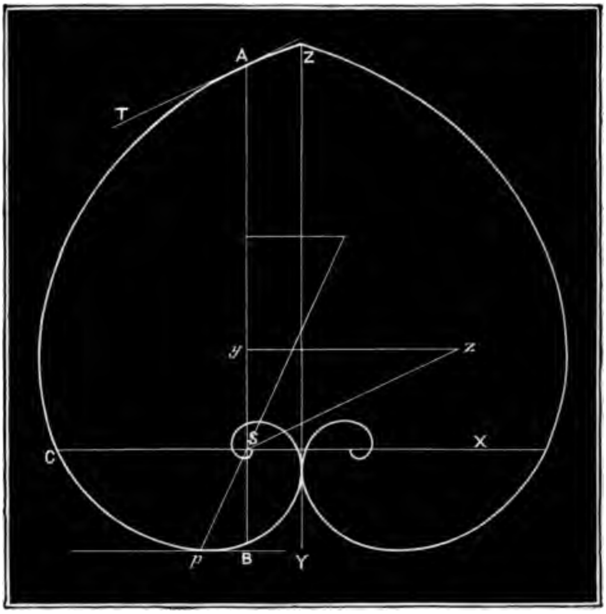
In volutes it is usual to let the innermost convolution of the spiral die away into a circular *eye* (fig. 160). To find the centre of this circle draw E P, L P, the normals at two consecutive points, meeting in P; then P is the centre, and P E the radius of the required circle which has a common tangent with the spiral at E, and P E is very nearly equal to the radius of curvature at E.

Fig. 160.



An application of this spiral to form the contour of a classical ornament is shown in figure 161, in which two equal spirals are turned reversed ways against each other, S being the focus, C S X the horizontal axis, A S B perpendicular thereto. In this case the ratio of S A to S B is that of 4 to 1, and the value of α, or the tangent of the angle between the radius-vector and tangent, is 2·2. Taking S y as unity on any scale, y z perpendicular thereto, and equal to 2·2 on the same scale, S z will be parallel to A T the tangent at A, and also to the tangent at B; z S y will represent the angle

Fig. 161.



between the tangent and radius-vector drawn from S at every point on the curve.

Make the angle B S p equal to the angle y z S, and p will be the lowest point on the curve, the tangent at p being parallel to C S X.

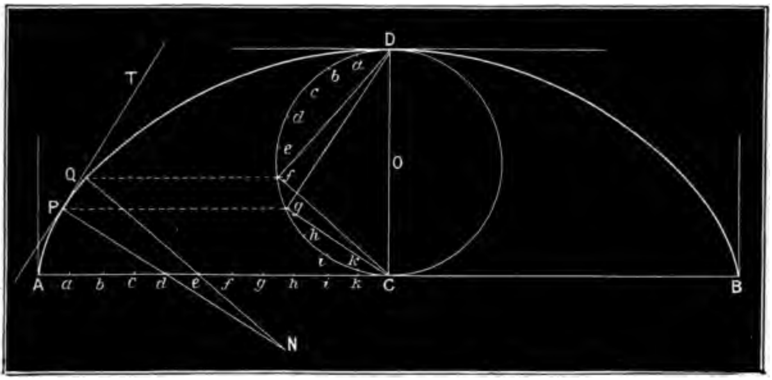
The equiangular-spiral being capable of any amount of variation, is the most valuable of all the known spirals for decorative purposes.

CHAPTER VI.

CYCLOIDS.

193. *To describe a cycloid by continuous motion.*—Let A B (fig. 162) be a straight line, as the edge of a ruler, and let a circle of given diameter C D roll along the line A B; then a pencil fixed at any point on the rolling circle, starting from A and returning to the straight line at B after having made one whole revolution, will mark out the curve A D B, called the *cycloid.*

Fig. 162.



194. *To find points upon the perimeter of the cycloid.*—Divide the line A C (fig. 162) into eleven equal parts, and also the circumference of the half-circle C D into eleven equal parts. Let C *g* be any number, as four, of those parts; draw *g* P parallel to A B, and take *g* P equal to A *g*, then P is a point on the cycloid. In the same way as many points can be found on the curve as there are divisions on the circle.

The part of the curve from D to B is exactly similar to the part from D to A.

195. *To draw the tangent and normal at any given point on the cycloid.*—Let P (fig. 162) be the given point on the curve. Draw P*g* parallel to A B cutting the generating circle in *g*; draw C*g* the chord of the circle, and at P draw P T parallel to D*g*, P T will be the required tangent. In the same way the tangent at any other point can be found by drawing a line parallel to the corresponding chord of the circle. The tangent at D is parallel to A B, and the tangents at A and B are parallel to the diameter C D, or at right-angles to A B.

Since the chord C*g* makes a right-angle with *g*D, the normal P N at the point P must be parallel to C*g*. In this way the normal can be drawn at any point without drawing the tangent. The diameter C D is the normal at the vertex D, and the line A B is the normal to the curve at A and B.

The normal P N can also be drawn by taking A*d* on A C equal to the arc C*g* on the generating circle, and drawing P N through the point *d* on A C.

To approximate to the cycloid by means of arcs of circles, draw the normals at P and Q, meeting in N, and from N as a centre draw the arc P Q. Proceed in the same manner at the other points found on the curve, and the arcs of circles drawn from the intersections of consecutive normals as centres will give a curve nearly resembling the cycloid. The radius of curvature has its greatest value at the vertex D where it is equal to twice the diameter C D of the generating circle; the curvature being least at the vertex

D and increasing towards A and B, at which points the radius of curvature is nothing. The length of the radius of curvature, P N, at any point P is equal to twice the chord C *g* of the circle.

196. *Properties and applications of the cycloid.*—The length of any arc as D P measured from the vertex D is equal to twice the length of the chord D *g* of the generating circle, and the whole length of the arc A D or B D is equal to twice the diameter C D. The area contained between the curve and its base line A B is equal to three times the area of the generating circle.

If we invert the cycloid so as to make its vertex D the lowest point, A B being kept horizontal, and a ball is allowed to fall from any point on the curve and constrained to descend in a groove having the form of the cycloid, the time of its descent to the lowest point will be the same from whatever point on the curve the descent commences; and if a pendulum is made to oscillate in a cycloid, its times of oscillation will be the same whatever the length of the arc it describes. The cycloid is also the curve of quickest descent of a body from one point to another not in the same vertical line.

The cycloid may be used as the contour of an arch, the height C D of which is to the span A B in the ratio of 7 to 22, or of 100 to 314. In this case the normals will represent the directions of the joints of the voussoirs of the arch.

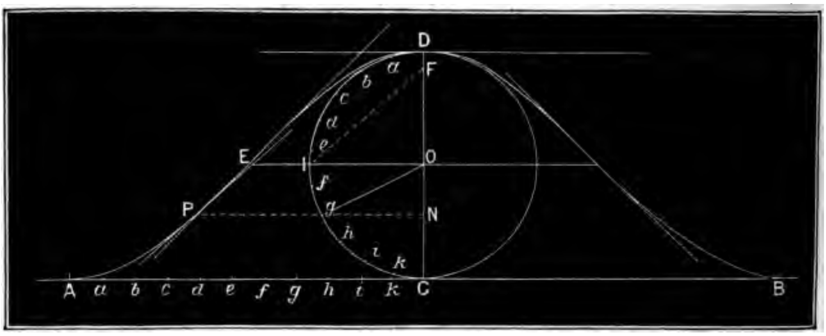
The cycloid is the proper form for the teeth of a *rack* which drives a cogged or pinion wheel, the part from A towards P being used for that purpose.

197. *To describe the curve called the companion to the cycloid.*—Let A B (fig. 163) represent the circumference of the circle whose diameter is C D, and its centre at O. Let

C be the middle point of A B, and divide A C into eleven equal parts, and also the circumference of the semicircle into the same number of equal parts. Draw any ordinate as N *g* P parallel to A C, and make N P equal to D *g* or A *g*, then P is a point on the required curve.

This curve has a point of contrary-flexure (**155**) at the ordinate O E equal to the quadrant of the circle. The tangent at E makes half a right-angle, or 45°, with the line E O. The tangent at D is parallel to A B, and at A and B the line A B is the tangent.

Fig. 163.



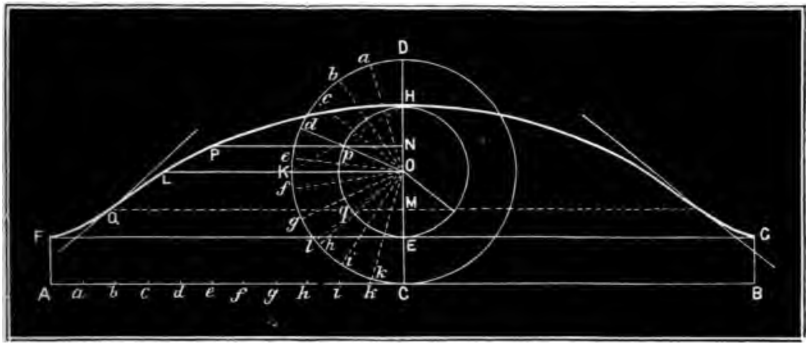
The tangent at any other point P can be drawn by taking O F equal to N *g*, and the line F I is parallel to the tangent at P.

This curve can be applied to form the contour of architectural mouldings, such as the ogee or sima, or for an ogival arch, of which C D is the half-span, A C the height.

198. DEFINITION.—When the pencil is placed at a point *within* the circumference of the generating circle the curve marked out is called a *trochoid.*

199. *To find points upon a trochoid.*—Let A B be the line on which rolls the generating circle whose diameter is C D (fig. 164), O *q* the distance from the centre of the pencil which describes the curve. Divide, as before, the line A C into eleven equal parts, and also the semicircle C K D. Draw the radii O *a*, O *b*, &c. Let the radius O *d* cut the inner circle in the point *p*, draw N *p* P parallel to A B, and take *p* P equal to the arc D *d*, or to the line A *d*; then P is a point on the trochoid. Let *m* represent the ratio of O E to O C, and make C M equal to the product of C O into the fraction $(1-m^2)$; draw M *q* Q parallel to A C cutting the inner circle at *q*. Join O *q* and produce it to meet the generating circle at *l*; take *q* Q equal to the arc D *l*, and the point Q will be one of contrary-flexure (**155**), and the tangent at Q is parallel to O *q*.

Fig. 164.



The curve has the line F G, parallel to A B, for its tangent at F and G; and the tangent at H is parallel to A B.

Draw O K L parallel to A B, meeting the curve at L, join H K, and the tangent at L will be parallel to H K.

200. *To describe an epicycloid and hypocycloid.*—Let the generating circle move outside the circumference of another circle, instead of on a straight-line as in the cycloid; then the curve which a pencil on the circumference of the rolling circle will mark out is called an *epicycloid.* This curve is used to form the outline of the teeth of two cogged wheels which roll upon the outside of each other.

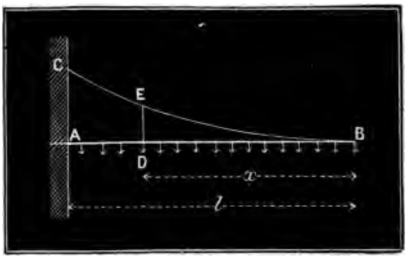
If the generating circle rolls on the inside of the circumference of another circle, then a point on its circumference marks out the *hypocycloid*, which is used to form the outline of the teeth of cogged wheels, one of which rolls on the inside of the other.

APPENDIX A.

DIAGRAMS OF STRAINS.

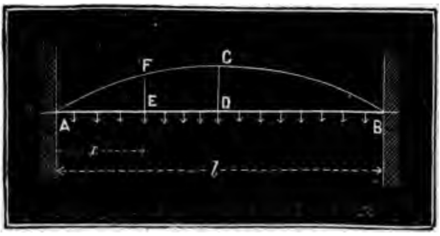
201. One of the most important applications of practical geometry to constructional purposes is the drawing of diagrams of strains, whereby the relative strain on each part of a frame can be seen at a glance, and we are thus enabled to determine the amount of strength necessary for the several parts of the structure. Suppose a beam A B (fig. 165) to be fixed in a wall at one end A, and to be loaded with a weight W, uniformly distributed over its entire length. Measure the length A C to represent on any convenient scale the quantity $\frac{1}{2}$ W × A B. Through the points B and C draw (**142**) a parabola having its vertex at B; then the *moment of strain* produced on the beam at any point D will be represented by the ordinate D E of the curve, on the same scale as that used for A C.

Fig. 165.



Let A B (fig. 166) be a beam supported at each end, and loaded uniformly throughout with a load W. Then if D is the centre of the beam, and we measure D C vertical and representing on a scale the quantity $\frac{1}{8}$ W × A B, and we draw a parabola through A, C and B,

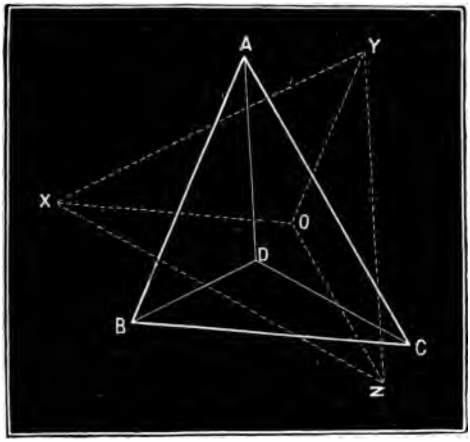
Fig. 166.



having its vertex at C, we find that the moment of strain at any point E of the beam will be represented on the same scale by the ordinate E F.

The strains upon the different parts of a frame can be determined by means of the following theorem propounded by the late Prof. Clerk Maxwell.

Fig. 167.



Let the triangle A B C (fig. 167) represent a frame jointed at the vertices, and kept in equilibrium by three forces acting at A, B, and C, the directions of which meet at the point D, and are proportional to the sides of a triangle parallel to the lines D A, D B, D C respectively, namely, the triangle X Y Z. Draw X O, Y O, Z O parallel to the sides of the frame, and these lines will represent the strains in those sides to which they are respectively parallel, to the same scale as the sides of the triangle X Y Z represent the original forces at A, B and C. The figure A B C is the 'frame-diagram,' and X Y Z is called the 'stress-diagram'; the lines which in the *frame* make a closed polygon are represented in the *stress-diagram* by lines meeting in a point; and, on the other hand, those which in the latter form a closed polygon, in the former are represented by lines which meet at a point; consequently the two are 'reciprocal figures.' By this means the stresses on each part of a frame can be determined by a geometrical process.

We will now apply this method to the case of a simple form of roof truss as A C B (fig. 168), which is the *frame-diagram*, the other figure (fig. 169) being the *stress-diagram*. Suppose W to be the total load upon the frame; then $\frac{1}{8}$ W is borne directly at A and at B, $\frac{1}{4}$ W at G, C and F. The vertical re-action at A and B is $\frac{1}{2}$ W.

Fig. 168.

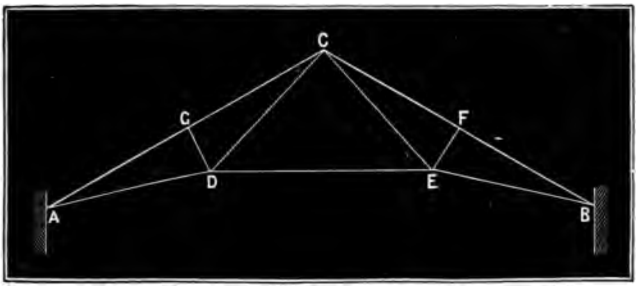
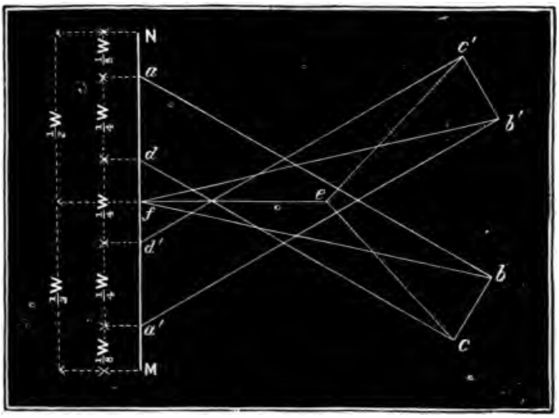


Fig. 169.

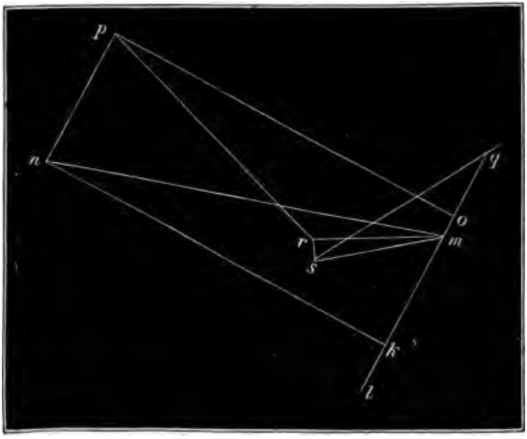


Draw the vertical M N (fig. 169) to represent W on any scale, and take f as its middle point. Let N a, M a', $f d$, $f d'$, be each taken equal to $\frac{1}{8}$ W on the above scale.

Draw *a b* and *d c* parallel to B C, *f b* parallel to B E, *b c* to E F, *c e* to E C, *f e* to D E. Also draw *a′ b′*, *d′ c′* parallel to A C, *f b′* to A D, *b′ c′* to D G, *c′ e* to D C. Then we can at once find, by using the above scale, the different strains in each of the parts of the frame, from the lengths of the lines parallel to them in the stress-diagram (fig. 169). Thus we have *f* N and *f* M for the re-action at A and B, N *a* and M *a′* for the loads supported at A and B, *a d* and *a′ d′* the vertical pressures at G and F, *d d′* that at C, *a b* and *a′ b′* the force down A G and B F, *d c* and *d′ c′* that along C G and C F, *b c* and *b′ c′* that in E F and G D, and *b f*, *b′ f* the strains in B E and A D; *e c* and *e c′* are the strains in E C and D C, and *f e* is that along D E.

Besides the vertical load W, there is, in the case of a roof truss, the pressure of the wind acting at one side only of the frame, and for which we require a separate *stress-diagram* (fig. 170). Suppose the pressure of the wind on the side B C to be represented by the load w acting at right angles to B C; then the pressure at each point B and C is $\frac{1}{4}w$, and $\frac{1}{2}w$ at F. Let the rafter B C make an angle of 30° with the horizontal; then the pressure at C produces a re-action at B of $\frac{1}{4}w \times \frac{1}{3}$ or $\frac{1}{12}w$, and of $\frac{1}{6}w$ at A. Also the pressure at F produces a re-action at B of $\frac{1}{2}w \times \frac{2}{3}$ or $\frac{1}{3}w$, and at A of $\frac{1}{2}w \times \frac{1}{3}$ or $\frac{1}{6}w$. So that the

Fig. 170.



total re-action at B is $\frac{1}{4}w+\frac{1}{12}w+\frac{1}{3}w$, or $\frac{2}{3}w$; and at A of $\frac{1}{6}w+\frac{1}{6}w$, or $\frac{1}{3}w$. Draw *q l* parallel to the direction of the pressure or at right-angles to B C, and take its length to represent the pressure *w*. Let *l k* and *q o* be each $\frac{1}{4}w$, take *l m* to represent the re-action at B, namely, $\frac{2}{3}w$. Draw *m n* parallel to B E and *k n* to B F; these lines will represent the strains in those beams respectively. Draw *o p* parallel to F C, and *n p* to E F; these will be the strains in those beams. Draw *q s* parallel to A C, and *m s* to A D, and we have the strains in those beams respectively. Lastly, draw *p r* parallel to C E, and *m r* to E D, and we have the strains in C E and E D respectively represented by these lines. Combining the strains from the two diagrams we obtain the total strain produced in each of the parts of the frame. By the same method we can easily find the relative strains in the several parts of any kind of frame whatever.

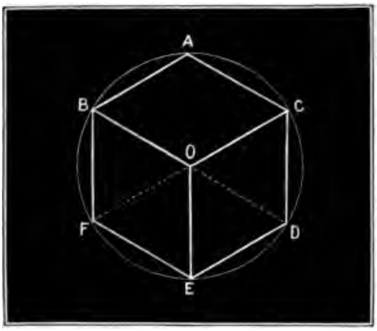
APPENDIX B.

ISOMETRICAL PROJECTION.

202. This method of projection is very useful when it is desired to represent a view of a group of buildings which cover a considerable space of ground; and, as all the parts are drawn to scale, we are able to measure them in the same way as in an ordinary plan or elevation.

Draw a circle having O for its centre (fig. 171), and in it describe a regular hexagon A B C D E F (**44**); draw the radii O B, O C, O E; then we shall have an 'isometrical' representation of a cube, the three visible planes of which, namely, A B O C, C D E O, B F E O, are all equal and similar, and all the boundary lines are also equal; hence the term 'isometrical' or 'equally measured' is applied to this kind of projection. The figure A B O C represents a horizontal square, and the two others represent vertical squares. The radii O B, O C, O E are termed *isometric axes*, all lines parallel to which are called *isometric lines*, O being the *regulating point*, and the three planes are called *isometric planes*.

Fig. 171.

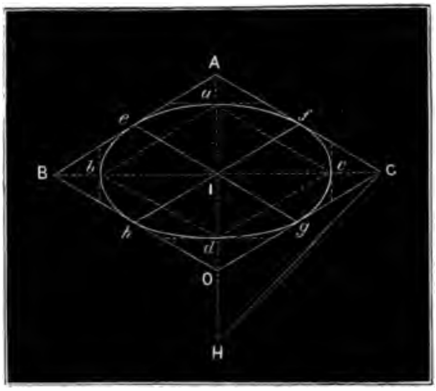


It is evident that all horizontal lines in the building will be represented in the projection by lines making the angle 30° with the horizontal, or 60° with the vertical, such as O B, E D, A B and A C, while all vertical lines in the building will also be vertical in the projection.

Since the *iso metric lines* are projections of the real lines of the building, they must be shorter in length, the proportion being as 1 to ·8165; so that if we have a scale of the building, or of its plan and elevation, showing 10,000 feet length, and we take 8,165 of those feet for another scale and divide it into 10,000, each of these divisions will represent a foot of measurement in the isometric projection, and with this scale we can measure any part of it vertically or horizontally.

The isometric representation of a horizontal square, such as A B O C (fig. 172), has the angles at A and O 120° each, and those at B and C 60° each; the diameter B C is the same as that of the original square, but the diameter A O is reduced in the proportion of tan. 30° to tan. 45°, or as 577 to 1,000. If we take I H = I C, then the line C H will be the side of the original square, of which C O is its isometrical projection.

Fig. 172.



If a circle is inscribed in the original square its isometrical projection will be an ellipse having its major axis *b c* equal to C H, and its minor axis $a\ d = \cdot577 \times \text{C H}$, so that the ellipse can be drawn as previously described (**105**), touching the sides of the projected square in their middle points *e*, *f*, *g* and *h*. The points *a* and *d* can also be found by drawing *b a* and *c d* parallel to A B. An octagon inscribed in the square will be projected in the same manner as the circle, *a d* being its diameter one way and *b c* its diameter at right angles to the former, the sides being found by drawing vertical lines through *b* and *c*, and horizontal ones through *a* and *d*, cutting the sides of the projected square, as shown by the dotted lines.

The above method of projection was first introduced by Prof. Farish for the purpose of representing models of machinery, and his paper on the subject will be found in Gregory's 'Mathematics for Practical Men.' *

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